A Logiweb test page

Klaus Grue

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Contents

1 Introduction

This paper tests some of the constructs introduced on the base and check pages.

2 Auxiliary definitions

$[\mathrm{tt} \stackrel{\mathrm{val}}{\rightarrow} \mathsf{T}]$

$[\mathrm{tf} \stackrel{\mathrm{val}}{\rightarrow} \mathsf{F}]$

$[\mathrm{ta} \stackrel{\mathrm{val}}{\rightarrow} \mathrm{tt} :: \mathrm{tt}]$

$[\mathrm{tb} \stackrel{\mathrm{val}}{\to} \mathrm{tt} :: \mathrm{tf}]$

$[\mathrm{tc} \stackrel{\mathrm{val}}{\to} \mathrm{tf} :: \mathrm{tt}]$

$[\mathrm{td} \stackrel{\mathrm{val}}{\to} \mathrm{tf} :: \mathrm{tf}]$

$[\mathrm{tx} \stackrel{\mathrm{val}}{\to} \bullet]$

$[\mathrm{ty} \stackrel{\mathrm{val}}{\to} 2^{\bullet}]$

$[\mathrm{tz} \stackrel{\mathrm{val}}{\to} 3^{\bullet}]$

$[\mathrm{tm} \stackrel{\mathrm{val}}{\to} \mathrm{map}\ (\ \lambda x.x\ )]$

$[\mathrm{to} \stackrel{\mathrm{val}}{\to} (\mathrm{Zero\ Pair\ Nine})\ \mathrm{Pair}\ 1]$

$[\mathrm{tO} \stackrel{\mathrm{val}}{\to} (\mathrm{One\ Pair\ Nine})\ \mathrm{Pair}\ 1]$

3 Test of test constructs

$[\mathsf{T}]^{\cdot}$

$[\mathsf{F}]^{-}$

$[\mathrm{tx} = \bullet]^{=}$

$[\mathrm{ty} = 2^{\bullet}]^{=}$

4 Untagged Booleans

4.1 Not

$[\mathrm{Not}\ \mathsf{T}]^{-}$

$[\mathrm{Not}\ \mathsf{F}]^{\cdot}$

4.2 And

$[\mathsf{T}\ \mathrm{And}\ \mathsf{T}]^{\cdot}$

$[\mathsf{T}\ \mathrm{And}\ \mathsf{F}]^{-}$

$[\mathsf{F}\ \mathrm{And}\ \mathsf{T}]^{-}$

$[\mathsf{F}\ \mathrm{And}\ \mathsf{F}]^{-}$

4.3 Or

[T Or T]$^{\cdot}$

[T Or F]$^{\cdot}$

[F Or T]$^{\cdot}$

[F Or F]$^{-}$

4.4 Iff

[T Iff T]$^{\cdot}$

[T Iff F]$^{-}$

[F Iff T]$^{-}$

[F Iff F]$^{\cdot}$

4.5 TheBool

[T TheBool]$^{\cdot}$

[F TheBool]$^{-}$

4.6 Equal

[T Equal T]$^{\cdot}$

[T Equal F]$^{-}$

[F Equal T]$^{-}$

[F Equal F]$^{\cdot}$

[T Pair T Equal T Pair T]$^{\cdot}$

[T Pair F Equal T Pair F]$^{\cdot}$

[F Pair T Equal F Pair T]$^{\cdot}$

[F Pair F Equal F Pair F]$^{\cdot}$

[F Pair T Equal T Pair T]$^{-}$

[T Pair F Equal T Pair T]$^{-}$

[T Pair T Equal F Pair T]$^{-}$

[T Pair T Equal T Pair F]$^{-}$

5 Untagged naturals

5.1 Small naturals

[Zero Equal T]$^{\cdot}$

[One Equal F Pair T]$^{\cdot}$

[Two Equal T Pair F Pair T]$^{\cdot}$

[Three Equal F Pair F Pair T]$^{\cdot}$

5.2 NatPair

[T NatPair Zero Equal Zero]$^{\cdot}$

[T NatPair One Equal Two]$^{\cdot}$

[Three NatPair T Equal One]$^{\cdot}$

[Three NatPair One Equal Three]$^{\cdot}$

5.3 TheNat

[(T) TheNat Equal Zero]$^{\cdot}$

[(T Pair T) TheNat Equal Zero]$^{\cdot}$

[(T Pair T Pair T) TheNat Equal Zero]$^{\cdot}$

[(T Pair T Pair T Pair T) TheNat Equal Zero]$^{\cdot}$

[(T Pair F Pair T Pair T Pair T) TheNat Equal Two]$^{\cdot}$

[(T Pair Three Pair T Pair T Pair T) TheNat Equal Two]$^{\cdot}$

5.4 Xor

[Xor (Two , Two , T)]$^{\cdot}$

[Xor (Two , Two , F)]$^{-}$

[Xor (Two , Three , T)]$^{-}$

[Xor (Two , Three , F)]$^{\cdot}$

[Xor (Three , Two , T)]$^{-}$

[Xor (Three , Two , F)]$^{\cdot}$

[Xor (Three , Three , T)]$^{\cdot}$

[Xor (Three , Three , F)]$^{-}$

5.5 Carry

[Carry (Two , Two , T)]$^{\cdot}$

[Carry (Two , Two , F)]$^{\cdot}$

[Carry (Two , Three , T)]$^{\cdot}$

[Carry (Two , Three , F)]$^{-}$

[Carry (Three , Two , T)]$^{\cdot}$

[Carry (Three , Two , F)]$^{-}$

[Carry (Three , Three , T)]$^{-}$

[Carry (Three , Three , F)]$^{-}$

5.6 Borrow

[Borrow (Two , Two , T)]$^{\cdot}$

[Borrow (Two , Two , F)]$^{-}$

[Borrow (Two , Three , T)]$^{-}$

[Borrow (Two , Three , F)]$^{-}$

[Borrow (Three , Two , T)]$^{\cdot}$

[Borrow (Three , Two , F)]$^{\cdot}$

[Borrow (Three , Three , T)]$^{\cdot}$

[Borrow (Three , Three , F)]$^{-}$

5.7 Plus

[Zero Plus Zero Equal Zero]$^{\cdot}$

[Zero Plus One Equal One]$^{\cdot}$

[Zero Plus Two Equal Two]$^{\cdot}$

[Zero Plus Three Equal Three]$^{\cdot}$

[One Plus Zero Equal One]$^{\cdot}$

[One Plus One Equal Two]$^{\cdot}$

[One Plus Two Equal Three]$^{\cdot}$

[One Plus Three Equal Four]$^{\cdot}$

[Two Plus Zero Equal Two].

[Two Plus One Equal Three].

[Two Plus Two Equal Four].

[Two Plus Three Equal Five].

[Three Plus Zero Equal Three].

[Three Plus One Equal Four].

[Three Plus Two Equal Five].

[Three Plus Three Equal Six].

5.8 LT

[Zero LT Zero Equal F].

[Zero LT One Equal T].

[Zero LT Two Equal T].

[Zero LT Three Equal T].

[One LT Zero Equal F].

[One LT One Equal F].

[One LT Two Equal T].

[One LT Three Equal T].

[Two LT Zero Equal F].

[Two LT One Equal F].

[Two LT Two Equal F].

[Two LT Three Equal T].

[Three LT Zero Equal F].

[Three LT One Equal F].

[Three LT Two Equal F].

[Three LT Three Equal F].

5.9 Minus

[Zero Minus Zero Equal Zero]˙

[One Minus Zero Equal One]˙

[One Minus One Equal Zero]˙

[Two Minus Zero Equal Two]˙

[Two Minus One Equal One]˙

[Two Minus Two Equal Zero]˙

[Three Minus Zero Equal Three]˙

[Three Minus One Equal Two]˙

[Three Minus Two Equal One]˙

[Three Minus Three Equal Zero]˙

5.10 Times

[Zero Times Zero Equal Zero]˙

[Zero Times One Equal Zero]˙

[Zero Times Two Equal Zero]˙

[Zero Times Three Equal Zero]˙

[One Times Zero Equal Zero]˙

[One Times One Equal One]˙

[One Times Two Equal Two]˙

[One Times Three Equal Three]˙

[Two Times Zero Equal Zero]˙

[Two Times One Equal Two]˙

[Two Times Two Equal Four]˙

[Two Times Three Equal Six]˙

[Three Times Zero Equal Zero]˙

[Three Times One Equal Three]˙

[Three Times Two Equal Six]˙

6 Booleans

6.1 boolp

$[\text{tt} \in \mathbf{B}]^{\cdot}$

$[\text{tf} \in \mathbf{B}]^{\cdot}$

$[1 \in \mathbf{B}]^{-}$

$[2 \in \mathbf{B}]^{-}$

$[^{-}2 \in \mathbf{B}]^{-}$

$[\text{ta} \in \mathbf{B}]^{-}$

$[\text{tb} \in \mathbf{B}]^{-}$

$[\text{tc} \in \mathbf{B}]^{-}$

$[\text{td} \in \mathbf{B}]^{-}$

$[\text{tx} = \text{tx} \in \mathbf{B}]^{=}$

$[\text{ty} = \text{ty} \in \mathbf{B}]^{=}$

$[\text{tm} \in \mathbf{B}]^{-}$

$[\text{to} \in \mathbf{B}]^{-}$

$[\text{tO} \in \mathbf{B}]^{-}$

6.2 truep

$[\text{tt} \in \mathbf{T}]^{\cdot}$

$[\text{tf} \in \mathbf{T}]^{-}$

$[1 \in \mathbf{T}]^{-}$

$[2 \in \mathbf{T}]^{-}$

$[^{-}2 \in \mathbf{T}]^{-}$

$[\text{ta} \in \mathbf{T}]^{-}$

$[\text{tb} \in \mathbf{T}]^{-}$

$[\text{tc} \in \mathbf{T}]^{-}$

$[\text{td} \in \mathbf{T}]^{-}$

$[\text{tx} = \text{tx} \in \mathbf{T}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \in \mathbf{T}]^{=}$

$[\mathrm{tm} \in \mathbf{T}]^{-}$

$[\mathrm{to} \in \mathbf{T}]^{-}$

$[\mathrm{tO} \in \mathbf{T}]^{-}$

6.3 falsep

$[\mathrm{tt} \in \mathbf{F}]^{-}$

$[\mathrm{tf} \in \mathbf{F}]^{\cdot}$

$[1 \in \mathbf{F}]^{-}$

$[2 \in \mathbf{F}]^{-}$

$[^{-}2 \in \mathbf{F}]^{-}$

$[\mathrm{ta} \in \mathbf{F}]^{-}$

$[\mathrm{tb} \in \mathbf{F}]^{-}$

$[\mathrm{tc} \in \mathbf{F}]^{-}$

$[\mathrm{td} \in \mathbf{F}]^{-}$

$[\mathrm{tx} = \mathrm{tx} \in \mathbf{F}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \in \mathbf{F}]^{=}$

$[\mathrm{tm} \in \mathbf{F}]^{-}$

$[\mathrm{to} \in \mathbf{F}]^{-}$

$[\mathrm{tO} \in \mathbf{F}]^{-}$

6.4 if-then-else

$[\mathbf{if}\ \mathrm{tt}\ \mathbf{then}\ \mathrm{tt}\ \mathbf{else}\ \mathrm{tt}]^{\cdot}$

$[\mathbf{if}\ \mathrm{tt}\ \mathbf{then}\ \mathrm{tt}\ \mathbf{else}\ \mathrm{tf}]^{\cdot}$

$[\mathbf{if}\ \mathrm{tt}\ \mathbf{then}\ \mathrm{tf}\ \mathbf{else}\ \mathrm{tt}]^{-}$

$[\mathbf{if}\ \mathrm{tt}\ \mathbf{then}\ \mathrm{tf}\ \mathbf{else}\ \mathrm{tf}]^{-}$

$[\mathbf{if}\ \mathrm{tf}\ \mathbf{then}\ \mathrm{tt}\ \mathbf{else}\ \mathrm{tt}]^{\cdot}$

$[\mathbf{if}\ \mathrm{tf}\ \mathbf{then}\ \mathrm{tt}\ \mathbf{else}\ \mathrm{tf}]^{-}$

$[\mathbf{if}\ \mathrm{tf}\ \mathbf{then}\ \mathrm{tf}\ \mathbf{else}\ \mathrm{tt}]^{\cdot}$

$[\textbf{if}\ \text{tf}\ \textbf{then}\ \text{tf}\ \textbf{else}\ \text{tf}]^{-}$

$[\textbf{if}\ \text{tf}\ \textbf{then}\ \text{tf}\ \textbf{else}\ \text{tf}]^{-}$

$[8 = \textbf{if}\ 1\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ 2\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ {}^{-}2\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ \text{ta}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ \text{tb}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ \text{tc}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ \text{td}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ \text{tm}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[\text{tx} = \textbf{if}\ \text{tx}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[\text{ty} = \textbf{if}\ \text{ty}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ \text{to}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

$[8 = \textbf{if}\ \text{tO}\ \textbf{then}\ 7\ \textbf{else}\ 8]^{=}$

6.5 not

$[\textbf{not}\ \text{tt}]^{-}$

$[\textbf{not}\ \text{tf}]^{\cdot}$

$[\textbf{not}\ 1]^{\cdot}$

$[\textbf{not}\ 2]^{\cdot}$

$[\textbf{not}\ {}^{-}2]^{\cdot}$

$[\textbf{not}\ \text{ta}]^{\cdot}$

$[\textbf{not}\ \text{tb}]^{\cdot}$

$[\textbf{not}\ \text{tc}]^{\cdot}$

$[\textbf{not}\ \text{td}]^{\cdot}$

$[\text{tx} = \textbf{not}\ \text{tx}]^{=}$

$[\text{ty} = \textbf{not}\ \text{ty}]^{=}$

$[\textbf{not}\ \text{tm}]^{\cdot}$

$[\textbf{not}\ \text{to}]^{\cdot}$

$[\textbf{not}\ \text{tO}]^{\cdot}$

6.6 notnot

$[\text{notnot tt}]^{\cdot}$

$[\text{notnot tf}]^{-}$

$[\text{notnot 1}]^{-}$

$[\text{notnot 2}]^{-}$

$[\text{notnot }^{-}2]^{-}$

$[\text{notnot ta}]^{-}$

$[\text{notnot tb}]^{-}$

$[\text{notnot tc}]^{-}$

$[\text{notnot td}]^{-}$

$[\text{tx} = \text{notnot tx}]^{=}$

$[\text{ty} = \text{notnot ty}]^{=}$

$[\text{notnot tm}]^{-}$

$[\text{notnot to}]^{-}$

$[\text{notnot tO}]^{-}$

6.7 and

$[\text{tt } \mathbf{and} \text{ tt}]^{\cdot}$

$[\text{tt } \mathbf{and} \text{ tf}]^{-}$

$[\text{tx} = \text{tt } \mathbf{and} \text{ tx}]^{=}$

$[\text{ty} = \text{tt } \mathbf{and} \text{ ty}]^{=}$

$[\text{tf } \mathbf{and} \text{ tt}]^{-}$

$[\text{tf } \mathbf{and} \text{ tf}]^{-}$

$[\text{tf } \mathbf{and} \text{ tx}]^{-}$

$[\text{tf } \mathbf{and} \text{ ty}]^{-}$

$[\text{tf } \mathbf{and} \perp]^{-}$

$[\text{tx} = \text{tx } \mathbf{and} \text{ tt}]^{=}$

$[\text{tx} = \text{tx } \mathbf{and} \text{ tf}]^{=}$

$[\text{tx} = \text{tx } \mathbf{and} \text{ tx}]^{=}$

$[\mathrm{tx} = \mathrm{tx}\ \mathbf{and}\ \mathrm{ty}]^{=}$

$[\mathrm{tx} = \mathrm{tx}\ \mathbf{and}\ \bot]^{=}$

$[\mathrm{ty} = \mathrm{ty}\ \mathbf{and}\ \mathrm{tt}]^{=}$

$[\mathrm{ty} = \mathrm{ty}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{ty} = \mathrm{ty}\ \mathbf{and}\ \mathrm{tx}]^{=}$

$[\mathrm{ty} = \mathrm{ty}\ \mathbf{and}\ \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty}\ \mathbf{and}\ \bot]^{=}$

$[1 = 1\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[2 = 2\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[^{-}2 = {}^{-}2\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{ta} = \mathrm{ta}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{tb} = \mathrm{tb}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{tc} = \mathrm{tc}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{td} = \mathrm{td}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{tm} = \mathrm{tm}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{to} = \mathrm{to}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[\mathrm{tO} = \mathrm{tO}\ \mathbf{and}\ \mathrm{tf}]^{=}$

$[1 = \mathrm{tt}\ \mathbf{and}\ 1]^{=}$

$[2 = \mathrm{tt}\ \mathbf{and}\ 2]^{=}$

$[^{-}2 = \mathrm{tt}\ \mathbf{and}\ {}^{-}2]^{=}$

$[\mathrm{ta} = \mathrm{tt}\ \mathbf{and}\ \mathrm{ta}]^{=}$

$[\mathrm{tb} = \mathrm{tt}\ \mathbf{and}\ \mathrm{tb}]^{=}$

$[\mathrm{tc} = \mathrm{tt}\ \mathbf{and}\ \mathrm{tc}]^{=}$

$[\mathrm{td} = \mathrm{tt}\ \mathbf{and}\ \mathrm{td}]^{=}$

$[\mathrm{tm} = \mathrm{tt}\ \mathbf{and}\ \mathrm{tm}]^{=}$

$[\mathrm{to} = \mathrm{tt}\ \mathbf{and}\ \mathrm{to}]^{=}$

$[\mathrm{tO} = \mathrm{tt}\ \mathbf{and}\ \mathrm{tO}]^{=}$

6.8 or

[tt **or** tt]$^{\cdot}$

[tt **or** tf]$^{\cdot}$

[tt **or** tx]$^{\cdot}$

[tf **or** tt]$^{\cdot}$

[tf **or** tf]$^{-}$

[tx = tf **or** tx]$^{=}$

[tx = tx **or** tt]$^{=}$

[tx = tx **or** tf]$^{=}$

[ty = ty **or** tx]$^{=}$

[8 = 1 **or** 8]$^{=}$

[8 = 2 **or** 8]$^{=}$

[8 = ¯2 **or** 8]$^{=}$

[8 = ta **or** 8]$^{=}$

[8 = tb **or** 8]$^{=}$

[8 = tc **or** 8]$^{=}$

[8 = td **or** 8]$^{=}$

[8 = tm **or** 8]$^{=}$

[8 = to **or** 8]$^{=}$

[8 = tO **or** 8]$^{=}$

[1 = 2 **or** 1]$^{=}$

[2 = ¯2 **or** 2]$^{=}$

[¯2 = ta **or** ¯2]$^{=}$

[ta = tb **or** ta]$^{=}$

[tb = tc **or** tb]$^{=}$

[tc = td **or** tc]$^{=}$

[td = tm **or** td]$^{=}$

[tm = to **or** tm]$^{=}$

[to = tO **or** to]$^{=}$

[tO = 1 **or** tO]$^{=}$

6.9 $x = y$

$[\mathrm{tt} = \mathrm{tt}]^{\cdot}$

$[\mathrm{tt} = \mathrm{tf}]^{-}$

$[\mathrm{tt} = 1]^{-}$

$[\mathrm{tt} = 2]^{-}$

$[\mathrm{tt} = {}^{-}2]^{-}$

$[\mathrm{tt} = \mathrm{ta}]^{-}$

$[\mathrm{tt} = \mathrm{tb}]^{-}$

$[\mathrm{tt} = \mathrm{tc}]^{-}$

$[\mathrm{tt} = \mathrm{td}]^{-}$

$[\mathrm{ty} = \mathrm{tt} = \mathrm{ty}]^{=}$

$[\mathrm{tt} = \mathrm{tm}]^{-}$

$[\mathrm{tt} = \mathrm{to}]^{-}$

$[\mathrm{tt} = \mathrm{tO}]^{-}$

$[\mathrm{tf} = \mathrm{tt}]^{-}$

$[\mathrm{tf} = \mathrm{tf}]^{\cdot}$

$[\mathrm{tf} = 1]^{-}$

$[\mathrm{tf} = 2]^{-}$

$[\mathrm{tf} = {}^{-}2]^{-}$

$[\mathrm{tf} = \mathrm{ta}]^{-}$

$[\mathrm{tf} = \mathrm{tb}]^{-}$

$[\mathrm{tf} = \mathrm{tc}]^{-}$

$[\mathrm{tf} = \mathrm{td}]^{-}$

$[\mathrm{ty} = \mathrm{tf} = \mathrm{ty}]^{=}$

$[\mathrm{tf} = \mathrm{tm}]^{-}$

$[\mathrm{tf} = \mathrm{to}]^{-}$

$[\mathrm{tf} = \mathrm{tO}]^{-}$

$[1 = \mathrm{tt}]^{-}$

$[1 = \text{tf}]^{-}$

$[1 = 1]^{\cdot}$

$[1 = 2]^{-}$

$[1 = {}^{-}2]^{-}$

$[1 = \text{ta}]^{-}$

$[1 = \text{tb}]^{-}$

$[1 = \text{tc}]^{-}$

$[1 = \text{td}]^{-}$

$[\text{ty} = 1 = \text{ty}]^{=}$

$[1 = \text{tm}]^{-}$

$[1 = \text{to}]^{-}$

$[1 = \text{tO}]^{-}$

$[2 = \text{tt}]^{-}$

$[2 = \text{tf}]^{-}$

$[2 = 1]^{-}$

$[2 = 2]^{\cdot}$

$[2 = {}^{-}2]^{-}$

$[2 = \text{ta}]^{-}$

$[2 = \text{tb}]^{-}$

$[2 = \text{tc}]^{-}$

$[2 = \text{td}]^{-}$

$[\text{ty} = 2 = \text{ty}]^{=}$

$[2 = \text{tm}]^{-}$

$[2 = \text{to}]^{-}$

$[2 = \text{tO}]^{-}$

$[{}^{-}2 = \text{tt}]^{-}$

$[{}^{-}2 = \text{tf}]^{-}$

$[{}^{-}2 = 1]^{-}$

$[{}^{-}2 = 2]^{-}$

$[{}^{-}2 = {}^{-}2]^{\cdot}$

$[{}^{-}2 = \mathrm{ta}]^{-}$

$[{}^{-}2 = \mathrm{tb}]^{-}$

$[{}^{-}2 = \mathrm{tc}]^{-}$

$[{}^{-}2 = \mathrm{td}]^{-}$

$[\mathrm{ty} = {}^{-}2 = \mathrm{ty}]^{=}$

$[{}^{-}2 = \mathrm{tm}]^{-}$

$[{}^{-}2 = \mathrm{to}]^{-}$

$[{}^{-}2 = \mathrm{tO}]^{-}$

$[\mathrm{ta} = \mathrm{tt}]^{-}$

$[\mathrm{ta} = \mathrm{tf}]^{-}$

$[\mathrm{ta} = 1]^{-}$

$[\mathrm{ta} = 2]^{-}$

$[\mathrm{ta} = {}^{-}2]^{-}$

$[\mathrm{ta} = \mathrm{ta}]^{\cdot}$

$[\mathrm{ta} = \mathrm{tb}]^{-}$

$[\mathrm{ta} = \mathrm{tc}]^{-}$

$[\mathrm{ta} = \mathrm{td}]^{-}$

$[\mathrm{ty} = \mathrm{ta} = \mathrm{ty}]^{=}$

$[\mathrm{ta} = \mathrm{tm}]^{-}$

$[\mathrm{ta} = \mathrm{to}]^{-}$

$[\mathrm{ta} = \mathrm{tO}]^{-}$

$[\mathrm{tb} = \mathrm{tt}]^{-}$

$[\mathrm{tb} = \mathrm{tf}]^{-}$

$[\mathrm{tb} = 1]^{-}$

$[\mathrm{tb} = 2]^{-}$

$[\mathrm{tb} = {}^{-}2]^{-}$

$[\mathrm{tb} = \mathrm{ta}]^-$

$[\mathrm{tb} = \mathrm{tb}]^\cdot$

$[\mathrm{tb} = \mathrm{tc}]^-$

$[\mathrm{tb} = \mathrm{td}]^-$

$[\mathrm{ty} = \mathrm{tb} = \mathrm{ty}]^=$

$[\mathrm{tb} = \mathrm{tm}]^-$

$[\mathrm{tb} = \mathrm{to}]^-$

$[\mathrm{tb} = \mathrm{tO}]^-$

$[\mathrm{tc} = \mathrm{tt}]^-$

$[\mathrm{tc} = \mathrm{tf}]^-$

$[\mathrm{tc} = 1]^-$

$[\mathrm{tc} = 2]^-$

$[\mathrm{tc} = {}^-2]^-$

$[\mathrm{tc} = \mathrm{ta}]^-$

$[\mathrm{tc} = \mathrm{tb}]^-$

$[\mathrm{tc} = \mathrm{tc}]^\cdot$

$[\mathrm{tc} = \mathrm{td}]^-$

$[\mathrm{ty} = \mathrm{tc} = \mathrm{ty}]^=$

$[\mathrm{tc} = \mathrm{tm}]^-$

$[\mathrm{tc} = \mathrm{to}]^-$

$[\mathrm{tc} = \mathrm{tO}]^-$

$[\mathrm{td} = \mathrm{tt}]^-$

$[\mathrm{td} = \mathrm{tf}]^-$

$[\mathrm{td} = 1]^-$

$[\mathrm{td} = 2]^-$

$[\mathrm{td} = {}^-2]^-$

$[\mathrm{td} = \mathrm{ta}]^-$

$[\mathrm{td} = \mathrm{tb}]^-$

$[\mathrm{td} = \mathrm{tc}]^{-}$

$[\mathrm{td} = \mathrm{td}]^{\cdot}$

$[\mathrm{ty} = \mathrm{td} = \mathrm{ty}]^{=}$

$[\mathrm{td} = \mathrm{tm}]^{-}$

$[\mathrm{td} = \mathrm{to}]^{-}$

$[\mathrm{td} = \mathrm{tO}]^{-}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{tt}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{tf}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = 1]^{=}$

$[\mathrm{ty} = \mathrm{ty} = 2]^{=}$

$[\mathrm{ty} = \mathrm{ty} = {}^{-}2]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{ta}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{tb}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{tc}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{td}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{tx}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{tm}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{to}]^{=}$

$[\mathrm{ty} = \mathrm{ty} = \mathrm{tO}]^{=}$

$[\mathrm{tm} = \mathrm{tt}]^{-}$

$[\mathrm{tm} = \mathrm{tf}]^{-}$

$[\mathrm{tm} = 1]^{-}$

$[\mathrm{tm} = 2]^{-}$

$[\mathrm{tm} = {}^{-}2]^{-}$

$[\mathrm{tm} = \mathrm{ta}]^{-}$

$[\mathrm{tm} = \mathrm{tb}]^{-}$

$[\mathrm{tm} = \mathrm{tc}]^{-}$

$[\mathrm{tm} = \mathrm{td}]^{-}$

$[\mathrm{ty} = \mathrm{tm} = \mathrm{ty}]^{=}$

$[\mathrm{tm} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tm} = \mathrm{to}]^{-}$

$[\mathrm{tm} = \mathrm{tO}]^{-}$

$[\mathrm{to} = \mathrm{tt}]^{-}$

$[\mathrm{to} = \mathrm{tf}]^{-}$

$[\mathrm{to} = 1]^{-}$

$[\mathrm{to} = 2]^{-}$

$[\mathrm{to} = {}^{-}2]^{-}$

$[\mathrm{to} = \mathrm{ta}]^{-}$

$[\mathrm{to} = \mathrm{tb}]^{-}$

$[\mathrm{to} = \mathrm{tc}]^{-}$

$[\mathrm{to} = \mathrm{td}]^{-}$

$[\mathrm{ty} = \mathrm{to} = \mathrm{ty}]^{=}$

$[\mathrm{to} = \mathrm{tm}]^{-}$

$[\mathrm{to} = \mathrm{to}]^{\cdot}$

$[\mathrm{to} = \mathrm{tO}]^{-}$

$[\mathrm{tO} = \mathrm{tt}]^{-}$

$[\mathrm{tO} = \mathrm{tf}]^{-}$

$[\mathrm{tO} = 1]^{-}$

$[\mathrm{tO} = 2]^{-}$

$[\mathrm{tO} = {}^{-}2]^{-}$

$[\mathrm{tO} = \mathrm{ta}]^{-}$

$[\mathrm{tO} = \mathrm{tb}]^{-}$

$[\mathrm{tO} = \mathrm{tc}]^{-}$

$[\mathrm{tO} = \mathrm{td}]^{-}$

$[\mathrm{ty} = \mathrm{tO} = \mathrm{ty}]^{=}$

$[\mathrm{tO} = \mathrm{tm}]^{-}$

$[\mathrm{tO} = \mathrm{to}]^{-}$

$[\mathrm{tO} = \mathrm{tO}]^{\cdot}$

$[((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb}))]^{\cdot}$

$[((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tc} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb}))]^{-}$

$[((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{tc}) :: (\mathrm{ta} :: \mathrm{tb}))]^{-}$

$[((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{tc} :: \mathrm{tb}))]^{-}$

$[((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tc}))]^{-}$

$[\mathrm{ty} = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{ty} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb}))]^{=}$

$[\mathrm{ty} = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{ty}) :: (\mathrm{ta} :: \mathrm{tb}))]^{=}$

$[\mathrm{ty} = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ty} :: \mathrm{tb}))]^{=}$

$[\mathrm{ty} = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{ty}))]^{=}$

$[\mathrm{ty} = ((\mathrm{tt} :: \mathrm{tf}) :: (\mathrm{ta} :: \mathrm{tb})) = ((\mathrm{tt} :: \mathrm{ty}) :: (\mathrm{tz} :: \mathrm{tb}))]^{=}$

6.10 $x \neq y$

$[\mathrm{tt} \neq \mathrm{tt}]^{-}$

$[\mathrm{tt} \neq \mathrm{tf}]^{\cdot}$

$[\mathrm{ty} = \mathrm{tt} \neq \mathrm{ty}]^{=}$

$[\mathrm{tf} \neq \mathrm{tt}]^{\cdot}$

$[\mathrm{tf} \neq \mathrm{tf}]^{-}$

$[\mathrm{ty} = \mathrm{tf} \neq \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \neq \mathrm{tt}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \neq \mathrm{tf}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \neq \mathrm{tx}]^{=}$

7 Integers

7.1 intp

$[\mathrm{tt} \in \mathbf{Z}]^{-}$

$[\mathrm{tf} \in \mathbf{Z}]^{-}$

$[1 \in \mathbf{Z}]^{\cdot}$

$[2 \in \mathbf{Z}]^{\cdot}$

$[{}^{-}2 \in \mathbf{Z}]^{\cdot}$

$[\mathrm{ta} \in \mathbf{Z}]^{-}$

$[\mathrm{tb} \in \mathbf{Z}]^{-}$

$[\mathrm{tc} \in \mathbf{Z}]^{-}$

$[\mathrm{td} \in \mathbf{Z}]^{-}$

$[\mathrm{ty} = \mathrm{ty} \in \mathbf{Z}]^{=}$

$[\mathrm{tm} \in \mathbf{Z}]^{-}$

$[\mathrm{to} \in \mathbf{Z}]^{-}$

$[\mathrm{tO} \in \mathbf{Z}]^{-}$

7.2 TheInt

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Zero}\ ) = \mathrm{TheInt}\ (\ \mathsf{F}\ ,\ \mathrm{Zero}\ )]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Zero}\ ) = 0]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{One}\ ) = 1]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Two}\ ) = 2]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Three}\ ) = 3]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Four}\ ) = 4]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Five}\ ) = 5]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Six}\ ) = 6]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Seven}\ ) = 7]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Eight}\ ) = 8]^{\cdot}$

$[\mathrm{TheInt}\ (\ \mathsf{T}\ ,\ \mathrm{Nine}\ ) = 9]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Zero}\ ) = -0]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{One}\ ) = {}^{-}1]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Two}\ ) = {}^{-}2]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Three}\ ) = {}^{-}3]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Four}\ ) = {}^{-}4]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Five}\ ) = {}^{-}5]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Six}\ ) = {}^{-}6]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Seven}\ ) = {}^{-}7]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Eight}\ ) = {}^{-}8]^{\cdot}$

$[\text{TheInt}\ (\ \mathsf{F}\ ,\ \text{Nine}\ ) = {}^{-}9]^{\cdot}$

7.3 PlusTag

$[\text{PlusTag}\ (\text{Zero}) = 0]^{\cdot}$

$[\text{PlusTag}\ (\text{One}) = 1]^{\cdot}$

$[\text{PlusTag}\ (\text{Two}) = 2]^{\cdot}$

$[\text{PlusTag}\ (\text{Three}) = 3]^{\cdot}$

$[\text{PlusTag}\ (\text{Four}) = 4]^{\cdot}$

$[\text{PlusTag}\ (\text{Five}) = 5]^{\cdot}$

$[\text{PlusTag}\ (\text{Six}) = 6]^{\cdot}$

$[\text{PlusTag}\ (\text{Seven}) = 7]^{\cdot}$

$[\text{PlusTag}\ (\text{Eight}) = 8]^{\cdot}$

$[\text{PlusTag}\ (\text{Nine}) = 9]^{\cdot}$

7.4 MinusTag

$[\text{MinusTag}\ (\text{Zero}) = -0]^{\cdot}$

$[\text{MinusTag}\ (\text{One}) = {}^{-}1]^{\cdot}$

$[\text{MinusTag}\ (\text{Two}) = {}^{-}2]^{\cdot}$

$[\text{MinusTag}\ (\text{Three}) = {}^{-}3]^{\cdot}$

$[\text{MinusTag}\ (\text{Four}) = {}^{-}4]^{\cdot}$

$[\text{MinusTag (Five)} = {}^{-}5]^{\cdot}$

$[\text{MinusTag (Six)} = {}^{-}6]^{\cdot}$

$[\text{MinusTag (Seven)} = {}^{-}7]^{\cdot}$

$[\text{MinusTag (Eight)} = {}^{-}8]^{\cdot}$

$[\text{MinusTag (Nine)} = {}^{-}9]^{\cdot}$

7.5 $x + y$

$[\text{tx} = \text{tt} + 1]^{=}$

$[\text{tx} = \text{tf} + 1]^{=}$

$[\text{tx} = \text{ta} + 1]^{=}$

$[\text{tx} = \text{tb} + 1]^{=}$

$[\text{tx} = \text{tc} + 1]^{=}$

$[\text{tx} = \text{td} + 1]^{=}$

$[\text{tx} = \text{tx} + 1]^{=}$

$[\text{ty} = \text{ty} + 1]^{=}$

$[\text{tx} = \text{tm} + 1]^{=}$

$[\text{tx} = \text{to} + 1]^{=}$

$[\text{tx} = \text{tO} + 1]^{=}$

$[\text{tx} = 1 + \text{tt}]^{=}$

$[\text{tx} = 1 + \text{tf}]^{=}$

$[\text{tx} = 1 + \text{ta}]^{=}$

$[\text{tx} = 1 + \text{tb}]^{=}$

$[\text{tx} = 1 + \text{tc}]^{=}$

$[\text{tx} = 1 + \text{td}]^{=}$

$[\text{tx} = 1 + \text{tx}]^{=}$

$[\text{ty} = 1 + \text{ty}]^{=}$

$[\text{tx} = 1 + \text{tm}]^{=}$

$[\text{tx} = 1 + \text{to}]^{=}$

$[\text{tx} = 1 + \text{tO}]^{=}$

$[\mathrm{ty} = \mathrm{ta} + \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty} + \mathrm{ta}]^{=}$

$[\mathrm{ty} = \mathrm{ty} + \bot]^{=}$

$[2 + 3 = 5]^{\cdot}$

$[2 + {}^{-}3 = {}^{-}1]^{\cdot}$

$[{}^{-}2 + 3 = 1]^{\cdot}$

$[{}^{-}2 + {}^{-}3 = {}^{-}5]^{\cdot}$

$[3 + 2 = 5]^{\cdot}$

$[{}^{-}3 + 2 = {}^{-}1]^{\cdot}$

$[3 + {}^{-}2 = 1]^{\cdot}$

$[{}^{-}3 + {}^{-}2 = {}^{-}5]^{\cdot}$

7.6 $x - y$

$[\mathrm{tx} = \mathrm{tt} - 1]^{=}$

$[\mathrm{tx} = \mathrm{tf} - 1]^{=}$

$[\mathrm{tx} = \mathrm{ta} - 1]^{=}$

$[\mathrm{tx} = \mathrm{tb} - 1]^{=}$

$[\mathrm{tx} = \mathrm{tc} - 1]^{=}$

$[\mathrm{tx} = \mathrm{td} - 1]^{=}$

$[\mathrm{tx} = \mathrm{tx} - 1]^{=}$

$[\mathrm{ty} = \mathrm{ty} - 1]^{=}$

$[\mathrm{tx} = \mathrm{tm} - 1]^{=}$

$[\mathrm{tx} = \mathrm{to} - 1]^{=}$

$[\mathrm{tx} = \mathrm{tO} - 1]^{=}$

$[\mathrm{tx} = 1 - \mathrm{tt}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{tf}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{ta}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{tb}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{tc}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{td}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{tx}]^{=}$

$[\mathrm{ty} = 1 - \mathrm{ty}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{tm}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{to}]^{=}$

$[\mathrm{tx} = 1 - \mathrm{tO}]^{=}$

$[\mathrm{ty} = \mathrm{ty} - \mathrm{ta}]^{=}$

$[\mathrm{ty} = \mathrm{ta} - \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty} - \bot]^{=}$

$[2 - 3 = {}^{-}1]^{\cdot}$

$[2 - {}^{-}3 = 5]^{\cdot}$

$[{}^{-}2 - 3 = {}^{-}5]^{\cdot}$

$[{}^{-}2 - {}^{-}3 = 1]^{\cdot}$

$[3 - 2 = 1]^{\cdot}$

$[{}^{-}3 - 2 = {}^{-}5]^{\cdot}$

$[3 - {}^{-}2 = 5]^{\cdot}$

$[{}^{-}3 - {}^{-}2 = {}^{-}1]^{\cdot}$

7.7 $x \cdot y$

$[\mathrm{tx} = \mathrm{tt} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{tf} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{ta} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{tb} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{tc} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{td} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{tx} \cdot 1]^{=}$

$[\mathrm{ty} = \mathrm{ty} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{tm} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{to} \cdot 1]^{=}$

$[\mathrm{tx} = \mathrm{tO} \cdot 1]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{tt}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{tf}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{ta}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{tb}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{tc}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{td}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{tx}]^{=}$

$[\mathrm{ty} = 1 \cdot \mathrm{ty}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{tm}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{to}]^{=}$

$[\mathrm{tx} = 1 \cdot \mathrm{tO}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \cdot \mathrm{ta}]^{=}$

$[\mathrm{ty} = \mathrm{ta} \cdot \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \cdot \perp]^{=}$

$[2 \cdot 3 = 6]^{\cdot}$

$[2 \cdot {}^{-}3 = {}^{-}6]^{\cdot}$

$[{}^{-}2 \cdot 3 = {}^{-}6]^{\cdot}$

$[{}^{-}2 \cdot {}^{-}3 = 6]^{\cdot}$

$[3 \cdot 2 = 6]^{\cdot}$

$[{}^{-}3 \cdot 2 = {}^{-}6]^{\cdot}$

$[3 \cdot {}^{-}2 = {}^{-}6]^{\cdot}$

$[{}^{-}3 \cdot {}^{-}2 = 6]^{\cdot}$

7.8 $x < y$

$[\text{tx} = \text{tt} < 1]^{=}$

$[\text{tx} = \text{tf} < 1]^{=}$

$[\text{tx} = \text{ta} < 1]^{=}$

$[\text{tx} = \text{tb} < 1]^{=}$

$[\text{tx} = \text{tc} < 1]^{=}$

$[\text{tx} = \text{td} < 1]^{=}$

$[\text{tx} = \text{tx} < 1]^{=}$

$[\text{ty} = \text{ty} < 1]^{=}$

$[\text{tx} = \text{tm} < 1]^{=}$

$[\text{tx} = \text{to} < 1]^{=}$

$[\text{tx} = \text{tO} < 1]^{=}$

$[\text{tx} = 1 < \text{tt}]^{=}$

$[\text{tx} = 1 < \text{tf}]^{=}$

$[\text{tx} = 1 < \text{ta}]^{=}$

$[\text{tx} = 1 < \text{tb}]^{=}$

$[\text{tx} = 1 < \text{tc}]^{=}$

$[\text{tx} = 1 < \text{td}]^{=}$

$[\text{tx} = 1 < \text{tx}]^{=}$

$[\text{ty} = 1 < \text{ty}]^{=}$

$[\text{tx} = 1 < \text{tm}]^{=}$

$[\text{tx} = 1 < \text{to}]^{=}$

$[\text{tx} = 1 < \text{tO}]^{=}$

$[\text{ty} = \text{ty} < \text{ta}]^{=}$

$[\text{ty} = \text{ta} < \text{ty}]^{=}$

$[\text{ty} = \text{ty} < \bot]^{=}$

$[2 < 3]^{\cdot}$

$[2 < {}^{-}3]^{-}$

$[{}^{-}2 < 3]^{\cdot}$

$[{}^{-}2 < {}^{-}3]^{-}$

$[3 < 2]^{-}$

$[{}^{-}3 < 2]^{\cdot}$

$[3 < {}^{-}2]^{-}$

$[{}^{-}3 < {}^{-}2]^{\cdot}$

$[2 < 2]^{-}$

$[{}^{-}2 < {}^{-}2]^{-}$

7.9 $x > y$

$[\mathrm{tx} = \mathrm{tt} > 1]^{=}$

$[\mathrm{tx} = \mathrm{tf} > 1]^{=}$

$[\mathrm{tx} = \mathrm{ta} > 1]^{=}$

$[\mathrm{tx} = \mathrm{tb} > 1]^{=}$

$[\mathrm{tx} = \mathrm{tc} > 1]^{=}$

$[\mathrm{tx} = \mathrm{td} > 1]^{=}$

$[\mathrm{tx} = \mathrm{tx} > 1]^{=}$

$[\mathrm{ty} = \mathrm{ty} > 1]^{=}$

$[\mathrm{tx} = \mathrm{tm} > 1]^{=}$

$[\mathrm{tx} = \mathrm{to} > 1]^{=}$

$[\mathrm{tx} = \mathrm{tO} > 1]^{=}$

$[\mathrm{tx} = 1 > \mathrm{tt}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{tf}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{ta}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{tb}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{tc}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{td}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{tx}]^{=}$

$[\mathrm{ty} = 1 > \mathrm{ty}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{tm}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{to}]^{=}$

$[\mathrm{tx} = 1 > \mathrm{tO}]^{=}$

$[\mathrm{ty} = \mathrm{ty} > \mathrm{ta}]^{=}$

$[\mathrm{ty} = \mathrm{ta} > \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty} > \bot]^{=}$

$[2 > 3]^{-}$

$[2 > {}^{-}3]^{\cdot}$

$[{}^{-}2 > 3]^{-}$

$[{}^{-}2 > {}^{-}3]^{\cdot}$

$[3 > 2]^{\cdot}$

$[{}^{-}3 > 2]^{-}$

$[3 > {}^{-}2]^{\cdot}$

$[{}^{-}3 > {}^{-}2]^{-}$

$[2 > 2]^{-}$

$[{}^{-}2 > {}^{-}2]^{-}$

7.10 $x \leq y$

$[\mathrm{tx} = \mathrm{tt} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{tf} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{ta} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{tb} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{tc} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{td} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{tx} \leq 1]^{=}$

$[\mathrm{ty} = \mathrm{ty} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{tm} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{to} \leq 1]^{=}$

$[\mathrm{tx} = \mathrm{tO} \leq 1]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{tt}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{tf}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{ta}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{tb}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{tc}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{td}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{tx}]^{=}$

$[\mathrm{ty} = 1 \leq \mathrm{ty}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{tm}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{to}]^{=}$

$[\mathrm{tx} = 1 \leq \mathrm{tO}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \leq \mathrm{ta}]^{=}$

$[\mathrm{ty} = \mathrm{ta} \leq \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \leq \bot]^{=}$

$[2 \leq 3]^{\cdot}$

$[2 \leq {}^{-}3]^{-}$

$[{}^{-}2 \leq 3]^{\cdot}$

$[{}^{-}2 \leq {}^{-}3]^{-}$

$[3 \leq 2]^{-}$

$[{}^{-}3 \leq 2]^{\cdot}$

$[3 \leq {}^{-}2]^{-}$

$[{}^{-}3 \leq {}^{-}2]^{\cdot}$

$[2 \leq 2]^{\cdot}$

$[{}^{-}2 \leq {}^{-}2]^{\cdot}$

7.11 $x \geq y$

$[\mathrm{tx} = \mathrm{tt} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{tf} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{ta} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{tb} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{tc} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{td} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{tx} \geq 1]^{=}$

$[\mathrm{ty} = \mathrm{ty} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{tm} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{to} \geq 1]^{=}$

$[\mathrm{tx} = \mathrm{tO} \geq 1]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{tt}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{tf}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{ta}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{tb}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{tc}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{td}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{tx}]^{=}$

$[\mathrm{ty} = 1 \geq \mathrm{ty}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{tm}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{to}]^{=}$

$[\mathrm{tx} = 1 \geq \mathrm{tO}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \geq \mathrm{ta}]^{=}$

$[\mathrm{ty} = \mathrm{ta} \geq \mathrm{ty}]^{=}$

$[\mathrm{ty} = \mathrm{ty} \geq \bot]^{=}$

$[2 \geq 3]^{-}$

$[2 \geq {}^{-}3]^{\cdot}$

$[{}^{-}2 \geq 3]^{-}$

$[{}^{-}2 \geq {}^{-}3]^{\cdot}$

$[3 \geq 2]^{\cdot}$

$[{}^{-}3 \geq 2]^{-}$

$[3 \geq {}^{-}2]^{\cdot}$

$[{}^{-}3 \geq {}^{-}2]^{-}$

$[2 \geq 2]^{\cdot}$

$[{}^{-}2 \geq {}^{-}2]^{\cdot}$

7.12 Strings

$[\mathrm{abc} \in \mathbf{Z}]^{\cdot}$

$[\mathrm{abc} = 97 + 256 \cdot 98 + 256 \cdot 256 \cdot 99 + 256 \cdot 256 \cdot 256]^{\cdot}$

$[\mathrm{if} = 91753]^{\cdot}$

8 Pairs

8.1 pairp

$[\mathrm{tt} \in \mathbf{P}]^{-}$

$[\mathrm{tf} \in \mathbf{P}]^{-}$

$[1 \in \mathbf{P}]^{-}$

$[2 \in \mathbf{P}]^{-}$

$[{}^{-}2 \in \mathbf{P}]^{-}$

$[\mathrm{ta} \in \mathbf{P}]^{\cdot}$

$[\mathrm{tb} \in \mathbf{P}]^{\cdot}$

$[\mathrm{tc} \in \mathbf{P}]^{\cdot}$

$[\mathrm{td} \in \mathbf{P}]^{\cdot}$

$[\mathrm{ty} = \mathrm{ty} \in \mathbf{P}]^{=}$

$[\mathrm{tm} \in \mathbf{P}]^{-}$

$[\mathrm{to} \in \mathbf{P}]^{-}$

$[\mathrm{tO} \in \mathbf{P}]^{-}$

8.2 head

$[\mathrm{tt} = \mathrm{tt}^{\mathrm{h}}]^{=}$

$[\mathrm{tf} = \mathrm{tf}^{\mathrm{h}}]^{=}$

$[1 = 1^{\mathrm{h}}]^{=}$

$[2 = 2^{\mathrm{h}}]^{=}$

$[{}^{-}2 = {}^{-}2^{\mathrm{h}}]^{=}$

$[\mathrm{ta}^{\mathrm{h}} = \mathrm{tt}]^{\cdot}$

$[\mathrm{tb}^{\mathrm{h}} = \mathrm{tt}]^{\cdot}$

$[\mathrm{tc}^{\mathrm{h}} = \mathrm{tf}]^{\cdot}$

$[\mathrm{td}^{\mathrm{h}} = \mathrm{tf}]^{\cdot}$

$[\mathrm{tx} = \mathrm{tx}^{\mathrm{h}}]^{=}$

$[\mathrm{ty} = \mathrm{ty}^{\mathrm{h}}]^{=}$

$[\mathrm{tm} = \mathrm{tm}^{\mathrm{h}}]^{=}$

$[\mathrm{to} = \mathrm{to}^{\mathrm{h}}]^{=}$

$[\mathrm{tO} = \mathrm{tO}^{\mathrm{h}}]^{=}$

$[(\mathrm{tt} :: 1)^{\mathrm{h}} = \mathrm{tt}]^{\cdot}$

$[(\mathrm{tf} :: 1)^{\mathrm{h}} = \mathrm{tf}]^{\cdot}$

$[(\mathrm{ta} :: 1)^{\mathrm{h}} = \mathrm{ta}]^{\cdot}$

$[(\mathrm{tb} :: 1)^{\mathrm{h}} = \mathrm{tb}]^{\cdot}$

$[(\mathrm{tc} :: 1)^{\mathrm{h}} = \mathrm{tc}]^{\cdot}$

$[(\mathrm{td} :: 1)^{\mathrm{h}} = \mathrm{td}]^{\cdot}$

$[\mathrm{ty} = (\mathrm{ty} :: 1)^{\mathrm{h}} = \mathrm{tt}]^{=}$

$[(\mathrm{tm} :: 1)^{\mathrm{h}} = \mathrm{tm}]^{\cdot}$

$[(\mathrm{to} :: 1)^{\mathrm{h}} = \mathrm{to}]^{\cdot}$

$[(\mathrm{tO} :: 1)^{\mathrm{h}} = \mathrm{tO}]^{\cdot}$

8.3 tail

$[\mathrm{tt} = \mathrm{tt}^{\mathrm{t}}]^{=}$

$[\mathrm{tf} = \mathrm{tf}^{\mathrm{t}}]^{=}$

$[1 = 1^{\mathrm{t}}]^{=}$

$[2 = 2^{\mathrm{t}}]^{=}$

$[{}^{-}2 = {}^{-}2^{\mathrm{t}}]^{=}$

$[\mathrm{ta}^{\mathrm{t}} = \mathrm{tt}]^{\cdot}$

$[\mathrm{tb}^{\mathrm{t}} = \mathrm{tf}]^{\cdot}$

$[\mathrm{tc}^{\mathrm{t}} = \mathrm{tt}]^{\cdot}$

$[\mathrm{td}^{\mathrm{t}} = \mathrm{tf}]^{\cdot}$

$[\mathrm{tx} = \mathrm{tx}^{\mathrm{t}}]^{=}$

$[\mathrm{ty} = \mathrm{ty}^{\mathrm{t}}]^{=}$

$[\mathrm{tm} = \mathrm{tm}^{\mathrm{t}}]^{=}$

$[\mathrm{to} = \mathrm{to}^{\mathrm{t}}]^{=}$

$[\mathrm{tO} = \mathrm{tO}^{\mathrm{t}}]^{=}$

$[(1 :: \mathrm{tt})^{\mathrm{t}} = \mathrm{tt}]^{\cdot}$

$[(1 :: \mathrm{tf})^{\mathrm{t}} = \mathrm{tf}]^{\cdot}$

$[(1 :: \mathrm{ta})^{\mathrm{t}} = \mathrm{ta}]^{\cdot}$

$[(1 :: \mathrm{tb})^{\mathrm{t}} = \mathrm{tb}]^{\cdot}$

$[(1 :: \mathrm{tc})^{\mathrm{t}} = \mathrm{tc}]^{\cdot}$

$[(1 :: \mathrm{td})^{\mathrm{t}} = \mathrm{td}]^{\cdot}$

$[\mathrm{ty} = (1 :: \mathrm{ty})^{\mathrm{t}} = \mathrm{tt}]^{=}$

$[(1 :: \mathrm{tm})^{\mathrm{t}} = \mathrm{tm}]^{\cdot}$

$[(1 :: \mathrm{to})^{\mathrm{t}} = \mathrm{to}]^{\cdot}$

$[(1 :: \mathrm{tO})^{\mathrm{t}} = \mathrm{tO}]^{\cdot}$

8.4 List operations

$[\text{revappend} \ (\ \langle 1,2,3 \rangle \ , \ \langle 4,5,6 \rangle \ ) = \langle 3,2,1,4,5,6 \rangle]^{=}$

$[\text{reverse} \ (\ \langle 1,2,3 \rangle \ ) = \langle 3,2,1 \rangle]^{=}$

$[\text{nth} \ (\ 2 \ , \ \langle 1,2,3,4 \rangle \ ) = 3]^{=}$

$[\text{length} \ (\ \langle 1,2,3 \rangle \ ) = 3]^{=}$

$[\text{list-prefix} \ (\ \langle 1,2,3,4 \rangle \ , \ 2 \ ) = \langle 1,2 \rangle]^{=}$

$[\text{list-suffix} \ (\ \langle 1,2,3,4 \rangle \ , \ 2 \ ) = \langle 3,4 \rangle]^{=}$

9 Exceptions

The following tests are merged from several sources. The merged test suite contains repetitions.

$[\bullet^{\circ \mathrm{h}}]^{\cdot}$

$[\mathsf{T}^{\circ \mathrm{h}}]^{-}$

$[\mathsf{F}^{\circ \mathrm{h}}]^{-}$

$[117^{\circ \mathrm{h}}]^{-}$

$[0^{\circ \mathrm{h}}]^{-}$

$[(-117)^{\circ \mathrm{h}}]^{-}$

$[(\mathsf{T} :: \mathsf{F})^{\circ \mathrm{h}}]^{-}$

$[\text{map} \ (\ \lambda x.x \ )^{\circ \mathrm{h}}]^{-}$

$[\bullet^{\circ \mathrm{t}} = \mathsf{T}]^{\cdot}$

$[\mathsf{T}^{\circ \mathrm{t}} = \mathsf{T}]^{\cdot}$

$[\mathsf{F}^{\circ \mathrm{t}} = \mathsf{F}]^{\cdot}$

$[117^{\circ \mathrm{t}} = 117]^{\cdot}$

$[0^{\circ \mathrm{t}} = 0]^{\cdot}$

$[(-117)^{\circ \mathrm{t}} = (-117)]^{\cdot}$

$[(\mathsf{T} :: \mathsf{F})^{\circ \mathrm{t}} = (\mathsf{T} :: \mathsf{F})]^{\cdot}$

$[\text{map} \ (\ \lambda x.x \ )^{\circ \mathrm{t}} = \text{map} \ (\ \lambda x.x \ )]^{\cdot}$

$[\bullet^{\circ} = \mathsf{T} :: \mathsf{T}]^{\cdot}$

$[\mathsf{T}^{\circ} = \mathsf{F} :: \mathsf{T}]^{\cdot}$

$[\mathsf{F}^\circ = \mathsf{F} :: \mathsf{F}]^\cdot$

$[117^\circ = \mathsf{F} :: 117]^\cdot$

$[0^\circ = \mathsf{F} :: 0]^\cdot$

$[(-117)^\circ = \mathsf{F} :: (-117)]^\cdot$

$[(\mathsf{T} :: \mathsf{F})^\circ = \mathsf{F} :: (\mathsf{T} :: \mathsf{F})]^\cdot$

$[\mathrm{map}\ (\ \lambda x.x\ )^\circ = \mathsf{F} :: \mathrm{map}\ (\ \lambda x.x\ )]^\cdot$

$[\mathrm{tt}^\circ = \mathsf{F} :: \mathrm{tt}]^\cdot$

$[\mathrm{tf}^\circ = \mathsf{F} :: \mathrm{tf}]^\cdot$

$[1^\circ = \mathsf{F} :: 1]^\cdot$

$[2^\circ = \mathsf{F} :: 2]^\cdot$

$[^{-}2^\circ = \mathsf{F} :: {}^{-}2]^\cdot$

$[\mathrm{ta}^\circ = \mathsf{F} :: \mathrm{ta}]^\cdot$

$[\mathrm{tb}^\circ = \mathsf{F} :: \mathrm{tb}]^\cdot$

$[\mathrm{tc}^\circ = \mathsf{F} :: \mathrm{tc}]^\cdot$

$[\mathrm{td}^\circ = \mathsf{F} :: \mathrm{td}]^\cdot$

$[\mathrm{tx}^\circ = \mathsf{T} :: \mathrm{tt}]^\cdot$

$[\mathrm{ty}^\circ = \mathsf{T} :: 2]^\cdot$

$[\mathrm{tz}^\circ = \mathsf{T} :: 3]^\cdot$

$[\mathrm{tm}^\circ = \mathsf{F} :: \mathrm{tm}]^\cdot$

$[\mathrm{to}^\circ = \mathsf{F} :: \mathrm{to}]^\cdot$

$[\mathrm{tO}^\circ = \mathsf{F} :: \mathrm{tO}]^\cdot$

$[2^{\bullet\circ} = \mathsf{T} :: 2]^\cdot$

$[2^{\bullet\bullet\circ} = \mathsf{T} :: 2]^\cdot$

10 Maps

10.1 mapp

$[\mathrm{tt} \in \mathbf{M}]^{-}$

$[\mathrm{tf} \in \mathbf{M}]^{-}$

$[1 \in \mathbf{M}]^{-}$

$[2 \in \mathbf{M}]^{-}$

$[{}^{-}2 \in \mathbf{M}]^{-}$

$[\mathrm{ta} \in \mathbf{M}]^{-}$

$[\mathrm{tb} \in \mathbf{M}]^{-}$

$[\mathrm{tc} \in \mathbf{M}]^{-}$

$[\mathrm{td} \in \mathbf{M}]^{-}$

$[\mathrm{ty} = \mathrm{ty} \in \mathbf{M}]^{=}$

$[\mathrm{tm} \in \mathbf{M}]^{\cdot}$

$[\mathrm{to} \in \mathbf{M}]^{-}$

$[\mathrm{tO} \in \mathbf{M}]^{-}$

10.2 map

$[\mathrm{map}\ (\ \mathrm{tt}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{tf}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ 1\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ 2\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ {}^{-}2\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{ta}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{tb}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{tc}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{td}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{tx}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{ty}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{tm}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{to}\ ) = \mathrm{tm}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{tO}\ ) = \mathrm{tm}]^{\cdot}$

10.3 catching maptag

$[\mathrm{tt}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tf}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[1^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[2^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[^{-}2^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{ta}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tb}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tc}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{td}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tx}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{ty}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tm}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{to}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tO}^{\mathrm{M}^\circ} = \mathrm{tm}]^{\cdot}$

10.4 maptag

$[\mathrm{tt}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tf}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[1^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[2^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[^{-}2^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{ta}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tb}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tc}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{td}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tx} = \mathrm{tx}^{\mathrm{M}} = \mathrm{tm}]^{=}$

$[\mathrm{ty} = \mathrm{ty}^{\mathrm{M}} = \mathrm{tm}]^{=}$

$[\mathrm{tm}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{to}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{tO}^{\mathrm{M}} = \mathrm{tm}]^{\cdot}$

10.5 untag

$[\mathrm{tx} = \mathrm{tt}^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = \mathrm{tf}^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = 1^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = 2^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = {}^{-}2^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = \mathrm{ta}^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = \mathrm{tb}^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = \mathrm{tc}^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = \mathrm{td}^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = \mathrm{tx}^{\mathrm{U}}]^{=}$

$[\mathrm{ty} = \mathrm{ty}^{\mathrm{U}}]^{=}$

$[\mathrm{tm}^{\mathrm{U}} = \mathrm{tf}]^{\cdot}$

$[\mathrm{tx} = \mathrm{to}^{\mathrm{U}}]^{=}$

$[\mathrm{tx} = \mathrm{tO}^{\mathrm{U}}]^{=}$

10.6 map+untag

$[\mathrm{map}\ (\ \mathrm{tt}\ )^{\mathrm{U}} = \mathrm{tt}]^{\cdot}$

$[\mathrm{map}\ (\ \mathrm{tf}\ )^{\mathrm{U}} = \mathrm{tf}]^{\cdot}$

$[\mathrm{map}\ (\ 1\ )^{\mathrm{U}} = 1]^{\cdot}$

$[\mathrm{map}\ (\ 2\ )^{\mathrm{U}} = 2]^{\cdot}$

$[\mathrm{map}\ (\ {}^{-}2\ )^{\mathrm{U}} = {}^{-}2]^{\cdot}$

$[\text{map (ta)}^{\text{U}} = \text{ta}]^{\cdot}$

$[\text{map (tb)}^{\text{U}} = \text{tb}]^{\cdot}$

$[\text{map (tc)}^{\text{U}} = \text{tc}]^{\cdot}$

$[\text{map (td)}^{\text{U}} = \text{td}]^{\cdot}$

$[\text{tx} = \text{map (tx)}^{\text{U}} = \text{tt}]^{=}$

$[\text{ty} = \text{map (ty)}^{\text{U}} = \text{tt}]^{=}$

$[\text{map (tm)}^{\text{U}} = \text{tm}]^{\cdot}$

$[\text{map (to)}^{\text{U}} = \text{to}]^{\cdot}$

$[\text{map (tO)}^{\text{U}} = \text{tO}]^{\cdot}$

10.7 catching maptag+untag

$[\text{tt}^{\text{M}^\circ\text{U}} = \text{tt}]^{\cdot}$

$[\text{tf}^{\text{M}^\circ\text{U}} = \text{tf}]^{\cdot}$

$[1^{\text{M}^\circ\text{U}} = 1]^{\cdot}$

$[2^{\text{M}^\circ\text{U}} = 2]^{\cdot}$

$[{}^{-}2^{\text{M}^\circ\text{U}} = {}^{-}2]^{\cdot}$

$[\text{ta}^{\text{M}^\circ\text{U}} = \text{ta}]^{\cdot}$

$[\text{tb}^{\text{M}^\circ\text{U}} = \text{tb}]^{\cdot}$

$[\text{tc}^{\text{M}^\circ\text{U}} = \text{tc}]^{\cdot}$

$[\text{td}^{\text{M}^\circ\text{U}} = \text{td}]^{\cdot}$

$[\text{tx} = \text{tx}^{\text{M}^\circ\text{U}} = \text{tt}]^{=}$

$[\text{ty} = \text{ty}^{\text{M}^\circ\text{U}} = \text{tt}]^{=}$

$[\text{tm}^{\text{M}^\circ\text{U}} = \text{tm}]^{\cdot}$

$[\text{to}^{\text{M}^\circ\text{U}} = \text{to}]^{\cdot}$

$[\text{tO}^{\text{M}^\circ\text{U}} = \text{tO}]^{\cdot}$

10.8 maptag+untag

$[\mathrm{tt}^{\mathrm{MU}} = \mathrm{tt}]^{\cdot}$

$[\mathrm{tf}^{\mathrm{MU}} = \mathrm{tf}]^{\cdot}$

$[1^{\mathrm{MU}} = 1]^{\cdot}$

$[2^{\mathrm{MU}} = 2]^{\cdot}$

$[{}^{-}2^{\mathrm{MU}} = {}^{-}2]^{\cdot}$

$[\mathrm{ta}^{\mathrm{MU}} = \mathrm{ta}]^{\cdot}$

$[\mathrm{tb}^{\mathrm{MU}} = \mathrm{tb}]^{\cdot}$

$[\mathrm{tc}^{\mathrm{MU}} = \mathrm{tc}]^{\cdot}$

$[\mathrm{td}^{\mathrm{MU}} = \mathrm{td}]^{\cdot}$

$[\mathrm{tx} = \mathrm{tx}^{\mathrm{MU}} = \mathrm{tt}]^{=}$

$[\mathrm{ty} = \mathrm{ty}^{\mathrm{MU}} = \mathrm{tt}]^{=}$

$[\mathrm{tm}^{\mathrm{MU}} = \mathrm{tm}]^{\cdot}$

$[\mathrm{to}^{\mathrm{MU}} = \mathrm{to}]^{\cdot}$

$[\mathrm{tO}^{\mathrm{MU}} = \mathrm{tO}]^{\cdot}$

10.9 root

$[\mathrm{tx} = \mathrm{tt}^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = \mathrm{tf}^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = 1^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = 2^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = {}^{-}2^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = \mathrm{ta}^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = \mathrm{tb}^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = \mathrm{tc}^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = \mathrm{td}^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = \mathrm{tx}^{\mathrm{R}}]^{=}$

$[\mathrm{ty} = \mathrm{ty}^{\mathrm{R}}]^{=}$

$[\mathrm{tm}^{\mathrm{R}}]^{-}$

$[\mathrm{tx} = \mathrm{to}^{\mathrm{R}}]^{=}$

$[\mathrm{tx} = \mathrm{tO}^{\mathrm{R}}]^{=}$

10.10 map+root

$[\text{map (tt)}^{\text{R}}]^{\cdot}$

$[\text{map (tf)}^{\text{R}}]^{-}$

$[\text{map (1)}^{\text{R}}]^{-}$

$[\text{map (2)}^{\text{R}}]^{-}$

$[\text{map (}^{-}\text{2)}^{\text{R}}]^{-}$

$[\text{map (ta)}^{\text{R}}]^{-}$

$[\text{map (tb)}^{\text{R}}]^{-}$

$[\text{map (tc)}^{\text{R}}]^{-}$

$[\text{map (td)}^{\text{R}}]^{-}$

$[\text{map (tx)}^{\text{R}}]^{-}$

$[\text{map (ty)}^{\text{R}}]^{-}$

$[\text{map (tm)}^{\text{R}}]^{-}$

$[\text{map (to)}^{\text{R}}]^{-}$

$[\text{map (tO)}^{\text{R}}]^{-}$

10.11 maptag+root

$[\text{tt}^{\text{MR}}]^{\cdot}$

$[\text{tf}^{\text{MR}}]^{-}$

$[\text{1}^{\text{MR}}]^{-}$

$[\text{2}^{\text{MR}}]^{-}$

$[^{-}\text{2}^{\text{MR}}]^{-}$

$[\text{ta}^{\text{MR}}]^{-}$

$[\text{tb}^{\text{MR}}]^{-}$

$[\text{tc}^{\text{MR}}]^{-}$

$[\text{td}^{\text{MR}}]^{-}$

$[\text{tx} = \text{tx}^{\text{MR}}]^{=}$

$[\text{ty} = \text{ty}^{\text{MR}}]^{=}$

$[\text{tm}^{\text{MR}}]^{-}$

$[\text{to}^{\text{MR}}]^{-}$

$[\text{tO}^{\text{MR}}]^{-}$

10.12 apply

$[(\mathrm{map}\ (\ \lambda x.x\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ ))^{\mathrm{R}}]^{\cdot}$

$[(\mathrm{map}\ (\ \lambda x.x\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ ))^{\mathrm{R}}]^{-}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.x\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ ))^{\mathrm{R}}]^{\cdot}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.x\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ ))^{\mathrm{R}}]^{\cdot}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.x\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ ))^{\mathrm{R}}]^{-}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.x\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ ))^{\mathrm{R}}]^{-}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.y\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ ))^{\mathrm{R}}]^{\cdot}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.y\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ ))^{\mathrm{R}}]^{-}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.y\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ )\ "\ \mathrm{map}\ (\ \mathrm{tt}\ ))^{\mathrm{R}}]^{\cdot}$

$[(\mathrm{map}\ (\ \lambda x.\lambda y.y\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ )\ "\ \mathrm{map}\ (\ \mathrm{tf}\ ))^{\mathrm{R}}]^{-}$

$[(\mathrm{map}\ (\ \lambda f.\lambda x.f\ '\ (f\ '\ x)\ )\ "\ \mathrm{map}\ (\ \lambda f.\lambda x.f\ '\ (f\ '\ x)\ )\ "\ \mathrm{map}\ (\ \lambda x.x+1\ )\ "\ \mathrm{map}\ (\ 2\ ))^{\mathrm{U}} = 6]^{\cdot}$

11 Logical operations on integers

11.1 half

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{tt}\ )]^{=}$

$[\mathrm{ty} = \mathrm{half}\ (\ \mathrm{ty}\ )]^{=}$

$[0 = \mathrm{half}\ (\ 0\ )]^{\cdot}$

$[0 = \mathrm{half}\ (\ 1\ )]^{\cdot}$

$[1 = \mathrm{half}\ (\ 2\ )]^{\cdot}$

$[1 = \mathrm{half}\ (\ 3\ )]^{\cdot}$

$[2 = \mathrm{half}\ (\ 4\ )]^{\cdot}$

$[2 = \mathrm{half}\ (\ 5\ )]^{\cdot}$

$[3 = \mathrm{half}\ (\ 6\ )]^{\cdot}$

$[3 = \mathrm{half}\ (\ 7\ )]^{\cdot}$

$[4 = \mathrm{half}\ (\ 8\ )]^{\cdot}$

$[4 = \mathrm{half}\ (\ 9\ )]^{\cdot}$

$[5 = \mathrm{half}\ (\ 10\ )]^{\cdot}$

$[{}^{-}1 = \mathrm{half}\ (\ {}^{-}1\ )]^{\cdot}$

$[{}^{-}1 = \mathrm{half}\ (\ {}^{-}2\ )]^{\cdot}$

$[{}^{-}2 = \mathrm{half}\ (\ {}^{-}3\ )]^{\cdot}$

$[{}^{-}2 = \mathrm{half}\ (\ {}^{-}4\ )]^{\cdot}$

$[{}^{-}3 = \mathrm{half}\ (\ {}^{-}5\ )]^{\cdot}$

$[{}^{-}3 = \mathrm{half}\ (\ {}^{-}6\ )]^{\cdot}$

$[{}^{-}4 = \mathrm{half}\ (\ {}^{-}7\ )]^{\cdot}$

$[{}^{-}4 = \mathrm{half}\ (\ {}^{-}8\ )]^{\cdot}$

$[{}^{-}5 = \mathrm{half}\ (\ {}^{-}9\ )]^{\cdot}$

$[{}^{-}5 = \mathrm{half}\ (\ -10\ )]^{\cdot}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{ta}\ )]^{=}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{tb}\ )]^{=}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{tc}\ )]^{=}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{td}\ )]^{=}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{tx}\ )]^{=}$

$[\mathrm{ty} = \mathrm{half}\ (\ \mathrm{ty}\ )]^{=}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{tm}\ )]^{=}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{to}\ )]^{=}$

$[\mathrm{tx} = \mathrm{half}\ (\ \mathrm{tO}\ )]^{=}$

11.2 evenp

$[\mathrm{tx} = \mathrm{evenp}\ (\ \mathrm{tt}\ )]^{=}$

$[\mathrm{ty} = \mathrm{evenp}\ (\ \mathrm{ty}\ )]^{=}$

$[\mathrm{evenp}\ (\ 0\ )]^{\cdot}$

$[\mathrm{evenp}\ (\ 1\ )]^{-}$

$[\mathrm{evenp}\ (\ 2\ )]^{\cdot}$

$[\mathrm{evenp}\ (\ 3\ )]^{-}$

$[\mathrm{evenp}\ (\ 4\ )]^{\cdot}$

$[\mathrm{evenp}\ (\ 5\ )]^{-}$

$[\text{evenp (6)}]^{\cdot}$

$[\text{evenp (7)}]^{-}$

$[\text{evenp (8)}]^{\cdot}$

$[\text{evenp (9)}]^{-}$

$[\text{evenp (10)}]^{\cdot}$

$[\text{evenp (}^{-}1\text{)}]^{-}$

$[\text{evenp (}^{-}2\text{)}]^{\cdot}$

$[\text{evenp (}^{-}3\text{)}]^{-}$

$[\text{evenp (}^{-}4\text{)}]^{\cdot}$

$[\text{evenp (}^{-}5\text{)}]^{-}$

$[\text{evenp (}^{-}6\text{)}]^{\cdot}$

$[\text{evenp (}^{-}7\text{)}]^{-}$

$[\text{evenp (}^{-}8\text{)}]^{\cdot}$

$[\text{evenp (}^{-}9\text{)}]^{-}$

$[\text{evenp (}-10\text{)}]^{\cdot}$

$[\text{tx} = \text{evenp (ta)}]^{=}$

$[\text{tx} = \text{evenp (tb)}]^{=}$

$[\text{tx} = \text{evenp (tc)}]^{=}$

$[\text{tx} = \text{evenp (td)}]^{=}$

$[\text{tx} = \text{evenp (tx)}]^{=}$

$[\text{ty} = \text{evenp (ty)}]^{=}$

$[\text{tx} = \text{evenp (tm)}]^{=}$

$[\text{tx} = \text{evenp (to)}]^{=}$

$[\text{tx} = \text{evenp (tO)}]^{=}$

11.3 small

[tx = small (tt)]$^{=}$

[tx = small (tf)]$^{=}$

[small (0)]$^{\cdot}$

[small (1)]$^{-}$

[small (2)]$^{-}$

[small (3)]$^{-}$

[small (4)]$^{-}$

[small (5)]$^{-}$

[small (6)]$^{-}$

[small (7)]$^{-}$

[small (8)]$^{-}$

[small (9)]$^{-}$

[small (10)]$^{-}$

[small ($^{-}$1)]$^{\cdot}$

[small ($^{-}$2)]$^{-}$

[small ($^{-}$3)]$^{-}$

[small ($^{-}$4)]$^{-}$

[small ($^{-}$5)]$^{-}$

[small ($^{-}$6)]$^{-}$

[small ($^{-}$7)]$^{-}$

[small ($^{-}$8)]$^{-}$

[small ($^{-}$9)]$^{-}$

[small ($-$10)]$^{-}$

[tx = small (ta)]$^{=}$

[tx = small (tb)]$^{=}$

[tx = small (tc)]$^{=}$

[tx = small (td)]$^{=}$

[tx = small (tx)]$^{=}$

[ty = small (ty)]$^{=}$

[tx = small (tm)]$^{=}$

[tx = small (to)]$^{=}$

[tx = small (tO)]$^{=}$

11.4 oddp

[tx = oddp (tt)]$^{=}$

[ty = oddp (ty)]$^{=}$

[oddp (6)]$^{-}$

[oddp (¯6)]$^{-}$

[oddp (7)]$^{\cdot}$

[oddp (¯7)]$^{\cdot}$

11.5 double

[tx = double (1 , 2)]$^{=}$

[tx = double (tt , tt)]$^{=}$

[ty = double (ty , $\perp$)]$^{=}$

[4 = double (tf , 2)]$^{\cdot}$

[5 = double (tt , 2)]$^{\cdot}$

[¯4 = double (tf , ¯2)]$^{\cdot}$

[¯3 = double (tt , ¯2)]$^{\cdot}$

11.6 lognot

[tx = lognot (tt)]$^{=}$

[ty = lognot (ty)]$^{=}$

[1 = lognot (¯2)]$^{\cdot}$

[0 = lognot (¯1)]$^{\cdot}$

[¯1 = lognot (0)]$^{\cdot}$

[¯2 = lognot (1)]$^{\cdot}$

11.7 logior

$[\mathrm{tx} = \mathrm{logior}\ (\ \mathrm{tt}\ ,\ 2\ )]^{=}$

$[\mathrm{tx} = \mathrm{logior}\ (\ 1\ ,\ \mathrm{tt}\ )]^{=}$

$[\mathrm{ty} = \mathrm{logior}\ (\ \mathrm{ty}\ ,\ \mathrm{tz}\ )]^{=}$

$[\mathrm{ty} = \mathrm{logior}\ (\ \mathrm{ty}\ ,\ \perp\ )]^{=}$

$[5 = \mathrm{logior}\ (\ 0\ ,\ 5\ )]^{\cdot}$

$[5 = \mathrm{logior}\ (\ 0\ ,\ 5\ )]^{\cdot}$

$[5 = \mathrm{logior}\ (\ 5\ ,\ 0\ )]^{\cdot}$

$[{}^{-}1 = \mathrm{logior}\ (\ {}^{-}1\ ,\ 5\ )]^{\cdot}$

$[{}^{-}1 = \mathrm{logior}\ (\ 5\ ,\ {}^{-}1\ )]^{\cdot}$

$[14 = \mathrm{logior}\ (\ 10\ ,\ 12\ )]^{\cdot}$

11.8 logxor

$[\mathrm{tx} = \mathrm{logxor}\ (\ \mathrm{tt}\ ,\ 2\ )]^{=}$

$[\mathrm{tx} = \mathrm{logxor}\ (\ 1\ ,\ \mathrm{tt}\ )]^{=}$

$[\mathrm{ty} = \mathrm{logxor}\ (\ \mathrm{ty}\ ,\ \perp\ )]^{=}$

$[5 = \mathrm{logxor}\ (\ 0\ ,\ 5\ )]^{\cdot}$

$[5 = \mathrm{logxor}\ (\ 5\ ,\ 0\ )]^{\cdot}$

$[{}^{-}6 = \mathrm{logxor}\ (\ {}^{-}1\ ,\ 5\ )]^{\cdot}$

$[{}^{-}6 = \mathrm{logxor}\ (\ 5\ ,\ {}^{-}1\ )]^{\cdot}$

$[6 = \mathrm{logxor}\ (\ 10\ ,\ 12\ )]^{\cdot}$

11.9 logand

$[\mathrm{tx} = \mathrm{logand}\ (\ \mathrm{tt}\ ,\ 2\ )]^{=}$

$[\mathrm{tx} = \mathrm{logand}\ (\ 1\ ,\ \mathrm{tt}\ )]^{=}$

$[\mathrm{ty} = \mathrm{logand}\ (\ \mathrm{ty}\ ,\ \perp\ )]^{=}$

$[0 = \mathrm{logand}\ (\ 0\ ,\ 5\ )]^{\cdot}$

$[0 = \mathrm{logand}\ (\ 5\ ,\ 0\ )]^{\cdot}$

$[5 = \mathrm{logand}\ (\ {}^{-}1\ ,\ 5\ )]^{\cdot}$

$[5 = \mathrm{logand}\ (\ 5\ ,\ {}^{-}1\ )]^{\cdot}$

$[8 = \mathrm{logand}\ (\ 10\ ,\ 12\ )]^{\cdot}$

11.10 logeqv

[tx = logeqv (tt , 2)]$^{=}$

[tx = logeqv (1 , tt)]$^{=}$

[ty = logeqv (ty , ⊥)]$^{=}$

[¯6 = logeqv (0 , 5)]$^{\cdot}$

[¯6 = logeqv (5 , 0)]$^{\cdot}$

[5 = logeqv (¯1 , 5)]$^{\cdot}$

[5 = logeqv (5 , ¯1)]$^{\cdot}$

[¯7 = logeqv (10 , 12)]$^{\cdot}$

11.11 lognand

[tx = lognand (tt , 2)]$^{=}$

[tx = lognand (1 , tt)]$^{=}$

[ty = lognand (ty , ⊥)]$^{=}$

[¯1 = lognand (0 , 5)]$^{\cdot}$

[¯1 = lognand (5 , 0)]$^{\cdot}$

[¯6 = lognand (¯1 , 5)]$^{\cdot}$

[¯6 = lognand (5 , ¯1)]$^{\cdot}$

[¯9 = lognand (10 , 12)]$^{\cdot}$

11.12 lognor

[tx = lognor (tt , 2)]$^{=}$

[tx = lognor (1 , tt)]$^{=}$

[ty = lognor (ty , ⊥)]$^{=}$

[¯6 = lognor (0 , 5)]$^{\cdot}$

[¯6 = lognor (5 , 0)]$^{\cdot}$

[0 = lognor (¯1 , 5)]$^{\cdot}$

[0 = lognor (5 , ¯1)]$^{\cdot}$

[¯15 = lognor (10 , 12)]$^{\cdot}$

11.13 logandc1

[tx = logandc1 (tt , 2)]$^{=}$

[tx = logandc1 (1 , tt)]$^{=}$

[ty = logandc1 (ty , ⊥)]$^{=}$

[5 = logandc1 (0 , 5)]$^{\cdot}$

[0 = logandc1 (5 , 0)]$^{\cdot}$

[0 = logandc1 (¯1 , 5)]$^{\cdot}$

[¯6 = logandc1 (5 , ¯1)]$^{\cdot}$

[4 = logandc1 (10 , 12)]$^{\cdot}$

11.14 logandc2

[tx = logandc2 (tt , 2)]$^{=}$

[tx = logandc2 (1 , tt)]$^{=}$

[ty = logandc2 (ty , ⊥)]$^{=}$

[0 = logandc2 (0 , 5)]$^{\cdot}$

[5 = logandc2 (5 , 0)]$^{\cdot}$

[¯6 = logandc2 (¯1 , 5)]$^{\cdot}$

[0 = logandc2 (5 , ¯1)]$^{\cdot}$

[2 = logandc2 (10 , 12)]$^{\cdot}$

11.15 logorc1

[tx = logorc1 (tt , 2)]$^{=}$

[tx = logorc1 (1 , tt)]$^{=}$

[ty = logorc1 (ty , ⊥)]$^{=}$

[¯1 = logorc1 (0 , 5)]$^{\cdot}$

[¯6 = logorc1 (5 , 0)]$^{\cdot}$

[5 = logorc1 (¯1 , 5)]$^{\cdot}$

[¯1 = logorc1 (5 , ¯1)]$^{\cdot}$

[¯3 = logorc1 (10 , 12)]$^{\cdot}$

11.16 logorc2

[tx = logorc2 (tt , 2)]$^{=}$

[tx = logorc2 (1 , tt)]$^{=}$

[ty = logorc2 (ty , ⊥)]$^{=}$

[¯6 = logorc2 (0 , 5)]$^{\cdot}$

[¯1 = logorc2 (5 , 0)]$^{\cdot}$

[¯1 = logorc2 (¯1 , 5)]$^{\cdot}$

[5 = logorc2 (5 , ¯1)]$^{\cdot}$

[¯5 = logorc2 (10 , 12)]$^{\cdot}$

11.17 logtest

[tx = logtest (tt , 2)]$^{=}$

[tx = logtest (1 , tt)]$^{=}$

[ty = logtest (ty , ⊥)]$^{=}$

[logtest (5 , 10)]$^{-}$

[logtest (5 , 0)]$^{-}$

[logtest (5 , 1)]$^{\cdot}$

[logtest (5 , 2)]$^{-}$

[logtest (5 , 4)]$^{\cdot}$

[logtest (5 , 8)]$^{-}$

[logtest (5 , 16)]$^{-}$

[logtest (¯11 , 10)]$^{-}$

[logtest (¯11 , 0)]$^{-}$

[logtest (¯11 , 1)]$^{\cdot}$

[logtest (¯11 , 2)]$^{-}$

[logtest (¯11 , 4)]$^{\cdot}$

[logtest (¯11 , 8)]$^{-}$

[logtest (¯11 , 16)]$^{\cdot}$

11.18 ash

[tx = ash (tt , 2)]$^{=}$

[tx = ash (1 , tt)]$^{=}$

[ty = ash (ty , ⊥)]$^{=}$

[3 = ash (3 , 0)]$^{\cdot}$

[6 = ash (3 , 1)]$^{\cdot}$

[12 = ash (3 , 2)]$^{\cdot}$

[13 = ash (13 , 0)]$^{\cdot}$

[6 = ash (13 , ¯1)]$^{\cdot}$

[3 = ash (13 , ¯2)]$^{\cdot}$

[1 = ash (13 , ¯3)]$^{\cdot}$

[0 = ash (13 , ¯4)]$^{\cdot}$

[0 = ash (13 , ¯5)]$^{\cdot}$

[¯3 = ash (¯3 , 0)]$^{\cdot}$

[¯6 = ash (¯3 , 1)]$^{\cdot}$

[¯12 = ash (¯3 , 2)]$^{\cdot}$

[¯6 = ash (¯6 , 0)]$^{\cdot}$

[¯3 = ash (¯6 , ¯1)]$^{\cdot}$

[¯2 = ash (¯6 , ¯2)]$^{\cdot}$

[¯1 = ash (¯6 , ¯3)]$^{\cdot}$

[¯1 = ash (¯6 , ¯4)]$^{\cdot}$

11.19 logbitp

[tx = logbitp (tt , 2)]$^{=}$

[tx = logbitp (1 , tt)]$^{=}$

[ty = logbitp (ty , ⊥)]$^{=}$

[logbitp (0 , 13)]$^{\cdot}$

[logbitp (1 , 13)]$^{-}$

[logbitp (2 , 13)]$^{\cdot}$

[logbitp (3 , 13)]$^{\cdot}$

[logbitp (4 , 13)]$^{-}$

[logbitp (5 , 13)]$^{-}$

[logbitp (6 , 13)]$^{-}$

[logbitp (0 , ¯6)]$^{-}$

[logbitp (1 , ¯6)]$^{\cdot}$

[logbitp (2 , ¯6)]$^{-}$

[logbitp (3 , ¯6)]$^{\cdot}$

[logbitp (4 , ¯6)]$^{\cdot}$

[logbitp (5 , ¯6)]$^{\cdot}$

[logbitp (6 , ¯6)]$^{\cdot}$

11.20 logcount

[tx = logcount (tt)]$^{=}$

[ty = logcount (ty)]$^{=}$

[0 = logcount (0)]$^{\cdot}$

[1 = logcount (1)]$^{\cdot}$

[1 = logcount (2)]$^{\cdot}$

[2 = logcount (3)]$^{\cdot}$

[1 = logcount (4)]$^{\cdot}$

[2 = logcount (5)]$^{\cdot}$

[2 = logcount (6)]$^{\cdot}$

[3 = logcount (7)]$^{\cdot}$

[1 = logcount (8)]$^{\cdot}$

[2 = logcount (9)]$^{\cdot}$

[0 = logcount (¯1)]$^{\cdot}$

[1 = logcount (¯2)]$^{\cdot}$

[1 = logcount (¯3)]$^{\cdot}$

[2 = logcount (¯4)]$^{\cdot}$

[1 = logcount (¯5)]$^{\cdot}$

[2 = logcount (¯6)]$^{\cdot}$

[2 = logcount (¯7)]$^{\cdot}$

[3 = logcount (¯8)]$^{\cdot}$

[1 = logcount (¯9)]$^{\cdot}$

11.21 integer-length

[tx = integer-length (tt)]$^{=}$

[ty = integer-length (ty)]$^{=}$

[0 = integer-length (0)]$^{\cdot}$

[1 = integer-length (1)]$^{\cdot}$

[2 = integer-length (2)]$^{\cdot}$

[2 = integer-length (3)]$^{\cdot}$

[3 = integer-length (4)]$^{\cdot}$

[3 = integer-length (5)]$^{\cdot}$

[3 = integer-length (6)]$^{\cdot}$

[3 = integer-length (7)]$^{\cdot}$

[4 = integer-length (8)]$^{\cdot}$

[4 = integer-length (9)]$^{\cdot}$

[0 = integer-length (¯1)]$^{\cdot}$

[1 = integer-length (¯2)]$^{\cdot}$

[2 = integer-length (¯3)]$^{\cdot}$

[2 = integer-length (¯4)]$^{\cdot}$

[3 = integer-length (¯5)]$^{\cdot}$

[3 = integer-length (¯6)]$^{\cdot}$

[3 = integer-length (¯7)]$^{\cdot}$

[3 = integer-length (¯8)]$^{\cdot}$

[4 = integer-length (¯9)]$^{\cdot}$

12 Numerals

$[\lceil 50 \rceil \stackrel{t}{=} \lceil 50 \rceil]^{\cdot}$

$[\lceil 51 \rceil \stackrel{t}{=} \lceil 51 \rceil]^{\cdot}$

$[\lceil 52 \rceil \stackrel{t}{=} \lceil 52 \rceil]^{\cdot}$

$[\lceil 53 \rceil \stackrel{t}{=} \lceil 53 \rceil]^{\cdot}$

$[\lceil 54 \rceil \stackrel{t}{=} \lceil 54 \rceil]^{\cdot}$

$[\lceil 55 \rceil \stackrel{t}{=} \lceil 55 \rceil]^{\cdot}$

$[\lceil 56 \rceil \stackrel{t}{=} \lceil 56 \rceil]^{\cdot}$

$[\lceil 57 \rceil \stackrel{t}{=} \lceil 57 \rceil]^{\cdot}$

$[\lceil 58 \rceil \stackrel{t}{=} \lceil 58 \rceil]^{\cdot}$

$[\lceil 59 \rceil \stackrel{t}{=} \lceil 59 \rceil]^{\cdot}$

$[\lceil 00 \rceil \stackrel{t}{=} \lceil 00 \rceil]^{\cdot}$

$[\lceil 20 \rceil \stackrel{t}{=} \lceil 20 \rceil]^{\cdot}$

$[\lceil 30 \rceil \stackrel{t}{=} \lceil 30 \rceil]^{\cdot}$

$[\lceil 40 \rceil \stackrel{t}{=} \lceil 40 \rceil]^{\cdot}$

$[\lceil 50 \rceil \stackrel{t}{=} \lceil 50 \rceil]^{\cdot}$

$[\lceil 60 \rceil \stackrel{t}{=} \lceil 60 \rceil]^{\cdot}$

$[\lceil 70 \rceil \stackrel{t}{=} \lceil 70 \rceil]^{\cdot}$

$[\lceil 80 \rceil \stackrel{t}{=} \lceil 80 \rceil]^{\cdot}$

$[\lceil 90 \rceil \stackrel{t}{=} \lceil 90 \rceil]^{\cdot}$

$[\lceil 12345678901 \rceil \stackrel{t}{=} \lceil 12345678901 \rceil]^{\cdot}$

$[\lceil 1x \rceil \stackrel{t}{=} \lceil \bullet \rceil]^{\cdot}$

$[123 + 45 = 168]^{\cdot}$

$[123 - 45 = 78]^{\cdot}$

$[123 \cdot 45 = 5535]^{\cdot}$

$[^{-}123 + {}^{-}45 = {}^{-}168]^{\cdot}$

$[^{-}123 - {}^{-}45 = {}^{-}78]^{\cdot}$

$[^{-}123 \cdot {}^{-}45 = 5535]^{\cdot}$

13 Division

13.1 floor

$[\text{floor}\ (\ {}^{-}9\ ,\ 1\ ) = {}^{-}9 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}8\ ,\ 1\ ) = {}^{-}8 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}7\ ,\ 1\ ) = {}^{-}7 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}6\ ,\ 1\ ) = {}^{-}6 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}5\ ,\ 1\ ) = {}^{-}5 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}4\ ,\ 1\ ) = {}^{-}4 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}3\ ,\ 1\ ) = {}^{-}3 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}2\ ,\ 1\ ) = {}^{-}2 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}1\ ,\ 1\ ) = {}^{-}1 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}0\ ,\ 1\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}1\ ,\ 1\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}2\ ,\ 1\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}3\ ,\ 1\ ) = {}^{+}3 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}4\ ,\ 1\ ) = {}^{+}4 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}5\ ,\ 1\ ) = {}^{+}5 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}6\ ,\ 1\ ) = {}^{+}6 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}7\ ,\ 1\ ) = {}^{+}7 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}8\ ,\ 1\ ) = {}^{+}8 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}9\ ,\ 1\ ) = {}^{+}9 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}9\ ,\ 2\ ) = {}^{-}5 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{-}8\ ,\ 2\ ) = {}^{-}4 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}7\ ,\ 2\ ) = {}^{-}4 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^-6\ ,\ 2\ ) = {}^-3 :: {}^+0]^=$

$[\text{floor}\ (\ {}^-5\ ,\ 2\ ) = {}^-3 :: {}^+1]^=$

$[\text{floor}\ (\ {}^-4\ ,\ 2\ ) = {}^-2 :: {}^+0]^=$

$[\text{floor}\ (\ {}^-3\ ,\ 2\ ) = {}^-2 :: {}^+1]^=$

$[\text{floor}\ (\ {}^-2\ ,\ 2\ ) = {}^-1 :: {}^+0]^=$

$[\text{floor}\ (\ {}^-1\ ,\ 2\ ) = {}^-1 :: {}^+1]^=$

$[\text{floor}\ (\ {}^+0\ ,\ 2\ ) = {}^+0 :: {}^+0]^=$

$[\text{floor}\ (\ {}^+1\ ,\ 2\ ) = {}^+0 :: {}^+1]^=$

$[\text{floor}\ (\ {}^+2\ ,\ 2\ ) = {}^+1 :: {}^+0]^=$

$[\text{floor}\ (\ {}^+3\ ,\ 2\ ) = {}^+1 :: {}^+1]^=$

$[\text{floor}\ (\ {}^+4\ ,\ 2\ ) = {}^+2 :: {}^+0]^=$

$[\text{floor}\ (\ {}^+5\ ,\ 2\ ) = {}^+2 :: {}^+1]^=$

$[\text{floor}\ (\ {}^+6\ ,\ 2\ ) = {}^+3 :: {}^+0]^=$

$[\text{floor}\ (\ {}^+7\ ,\ 2\ ) = {}^+3 :: {}^+1]^=$

$[\text{floor}\ (\ {}^+8\ ,\ 2\ ) = {}^+4 :: {}^+0]^=$

$[\text{floor}\ (\ {}^+9\ ,\ 2\ ) = {}^+4 :: {}^+1]^=$

$[\text{floor}\ (\ {}^-9\ ,\ 3\ ) = {}^-3 :: {}^+0]^=$

$[\text{floor}\ (\ {}^-8\ ,\ 3\ ) = {}^-3 :: {}^+1]^=$

$[\text{floor}\ (\ {}^-7\ ,\ 3\ ) = {}^-3 :: {}^+2]^=$

$[\text{floor}\ (\ {}^-6\ ,\ 3\ ) = {}^-2 :: {}^+0]^=$

$[\text{floor}\ (\ {}^-5\ ,\ 3\ ) = {}^-2 :: {}^+1]^=$

$[\text{floor}\ (\ {}^-4\ ,\ 3\ ) = {}^-2 :: {}^+2]^=$

$[\text{floor}\ (\ {}^-3\ ,\ 3\ ) = {}^-1 :: {}^+0]^=$

$[\text{floor}\ (\ {}^-2\ ,\ 3\ ) = {}^-1 :: {}^+1]^=$

$[\text{floor}\ (\ {}^-1\ ,\ 3\ ) = {}^-1 :: {}^+2]^=$

$[\text{floor}\ (\ {}^+0\ ,\ 3\ ) = {}^+0 :: {}^+0]^=$

$[\text{floor}\ (\ {}^+1\ ,\ 3\ ) = {}^+0 :: {}^+1]^=$

$[\text{floor}\ (\ {}^+2\ ,\ 3\ ) = {}^+0 :: {}^+2]^=$

$[\text{floor}\ (\ {}^{+}3\ ,\ 3\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}4\ ,\ 3\ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{+}5\ ,\ 3\ ) = {}^{+}1 :: {}^{+}2]^{=}$

$[\text{floor}\ (\ {}^{+}6\ ,\ 3\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}7\ ,\ 3\ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{+}8\ ,\ 3\ ) = {}^{+}2 :: {}^{+}2]^{=}$

$[\text{floor}\ (\ {}^{+}9\ ,\ 3\ ) = {}^{+}3 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}9\ ,\ 4\ ) = {}^{-}3 :: {}^{+}3]^{=}$

$[\text{floor}\ (\ {}^{-}8\ ,\ 4\ ) = {}^{-}2 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}7\ ,\ 4\ ) = {}^{-}2 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{-}6\ ,\ 4\ ) = {}^{-}2 :: {}^{+}2]^{=}$

$[\text{floor}\ (\ {}^{-}5\ ,\ 4\ ) = {}^{-}2 :: {}^{+}3]^{=}$

$[\text{floor}\ (\ {}^{-}4\ ,\ 4\ ) = {}^{-}1 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{-}3\ ,\ 4\ ) = {}^{-}1 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{-}2\ ,\ 4\ ) = {}^{-}1 :: {}^{+}2]^{=}$

$[\text{floor}\ (\ {}^{-}1\ ,\ 4\ ) = {}^{-}1 :: {}^{+}3]^{=}$

$[\text{floor}\ (\ {}^{+}0\ ,\ 4\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}1\ ,\ 4\ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{+}2\ ,\ 4\ ) = {}^{+}0 :: {}^{+}2]^{=}$

$[\text{floor}\ (\ {}^{+}3\ ,\ 4\ ) = {}^{+}0 :: {}^{+}3]^{=}$

$[\text{floor}\ (\ {}^{+}4\ ,\ 4\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}5\ ,\ 4\ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{+}6\ ,\ 4\ ) = {}^{+}1 :: {}^{+}2]^{=}$

$[\text{floor}\ (\ {}^{+}7\ ,\ 4\ ) = {}^{+}1 :: {}^{+}3]^{=}$

$[\text{floor}\ (\ {}^{+}8\ ,\ 4\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{floor}\ (\ {}^{+}9\ ,\ 4\ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{-}9\ ,\ 5\ ) = {}^{-}2 :: {}^{+}1]^{=}$

$[\text{floor}\ (\ {}^{-}8\ ,\ 5\ ) = {}^{-}2 :: {}^{+}2]^{=}$

$[\text{floor} \ (\ ^{-}7 \ , \ 5 \ ) = {^{-}2} :: {^{+}3}]^{=}$

$[\text{floor} \ (\ ^{-}6 \ , \ 5 \ ) = {^{-}2} :: {^{+}4}]^{=}$

$[\text{floor} \ (\ ^{-}5 \ , \ 5 \ ) = {^{-}1} :: {^{+}0}]^{=}$

$[\text{floor} \ (\ ^{-}4 \ , \ 5 \ ) = {^{-}1} :: {^{+}1}]^{=}$

$[\text{floor} \ (\ ^{-}3 \ , \ 5 \ ) = {^{-}1} :: {^{+}2}]^{=}$

$[\text{floor} \ (\ ^{-}2 \ , \ 5 \ ) = {^{-}1} :: {^{+}3}]^{=}$

$[\text{floor} \ (\ ^{-}1 \ , \ 5 \ ) = {^{-}1} :: {^{+}4}]^{=}$

$[\text{floor} \ (\ ^{+}0 \ , \ 5 \ ) = {^{+}0} :: {^{+}0}]^{=}$

$[\text{floor} \ (\ ^{+}1 \ , \ 5 \ ) = {^{+}0} :: {^{+}1}]^{=}$

$[\text{floor} \ (\ ^{+}2 \ , \ 5 \ ) = {^{+}0} :: {^{+}2}]^{=}$

$[\text{floor} \ (\ ^{+}3 \ , \ 5 \ ) = {^{+}0} :: {^{+}3}]^{=}$

$[\text{floor} \ (\ ^{+}4 \ , \ 5 \ ) = {^{+}0} :: {^{+}4}]^{=}$

$[\text{floor} \ (\ ^{+}5 \ , \ 5 \ ) = {^{+}1} :: {^{+}0}]^{=}$

$[\text{floor} \ (\ ^{+}6 \ , \ 5 \ ) = {^{+}1} :: {^{+}1}]^{=}$

$[\text{floor} \ (\ ^{+}7 \ , \ 5 \ ) = {^{+}1} :: {^{+}2}]^{=}$

$[\text{floor} \ (\ ^{+}8 \ , \ 5 \ ) = {^{+}1} :: {^{+}3}]^{=}$

$[\text{floor} \ (\ ^{+}9 \ , \ 5 \ ) = {^{+}1} :: {^{+}4}]^{=}$

$[\text{floor} \ (\ ^{-}5 \ , \ 0 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{-}1 \ , \ 0 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{+}0 \ , \ 0 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{+}1 \ , \ 0 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{+}5 \ , \ 0 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{-}5 \ , \ ^{-}1 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{-}1 \ , \ ^{-}1 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{+}0 \ , \ ^{-}1 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{+}1 \ , \ ^{-}1 \ ) = \bullet]^{=}$

$[\text{floor} \ (\ ^{+}5 \ , \ ^{-}1 \ ) = \bullet]^{=}$

13.2 div

$[{}^{-}9 \text{ div } 1 = {}^{-}9]^{=}$

$[{}^{-}8 \text{ div } 1 = {}^{-}8]^{=}$

$[{}^{-}7 \text{ div } 1 = {}^{-}7]^{=}$

$[{}^{-}6 \text{ div } 1 = {}^{-}6]^{=}$

$[{}^{-}5 \text{ div } 1 = {}^{-}5]^{=}$

$[{}^{-}4 \text{ div } 1 = {}^{-}4]^{=}$

$[{}^{-}3 \text{ div } 1 = {}^{-}3]^{=}$

$[{}^{-}2 \text{ div } 1 = {}^{-}2]^{=}$

$[{}^{-}1 \text{ div } 1 = {}^{-}1]^{=}$

$[{}^{+}0 \text{ div } 1 = {}^{+}0]^{=}$

$[{}^{+}1 \text{ div } 1 = {}^{+}1]^{=}$

$[{}^{+}2 \text{ div } 1 = {}^{+}2]^{=}$

$[{}^{+}3 \text{ div } 1 = {}^{+}3]^{=}$

$[{}^{+}4 \text{ div } 1 = {}^{+}4]^{=}$

$[{}^{+}5 \text{ div } 1 = {}^{+}5]^{=}$

$[{}^{+}6 \text{ div } 1 = {}^{+}6]^{=}$

$[{}^{+}7 \text{ div } 1 = {}^{+}7]^{=}$

$[{}^{+}8 \text{ div } 1 = {}^{+}8]^{=}$

$[{}^{+}9 \text{ div } 1 = {}^{+}9]^{=}$

$[{}^{-}9 \text{ div } 2 = {}^{-}5]^{=}$

$[{}^{-}8 \text{ div } 2 = {}^{-}4]^{=}$

$[{}^{-}7 \text{ div } 2 = {}^{-}4]^{=}$

$[{}^{-}6 \text{ div } 2 = {}^{-}3]^{=}$

$[{}^{-}5 \text{ div } 2 = {}^{-}3]^{=}$

$[{}^{-}4 \text{ div } 2 = {}^{-}2]^{=}$

$[{}^{-}3 \text{ div } 2 = {}^{-}2]^{=}$

$[{}^{-}2 \text{ div } 2 = {}^{-}1]^{=}$

$[{}^{-}1 \text{ div } 2 = {}^{-}1]^{=}$

$[{}^{+}0 \text{ div } 2 = {}^{+}0]^{=}$

$[{}^{+}1 \text{ div } 2 = {}^{+}0]^{=}$

$[{}^{+}2 \text{ div } 2 = {}^{+}1]^{=}$

$[{}^{+}3 \text{ div } 2 = {}^{+}1]^{=}$

$[{}^{+}4 \text{ div } 2 = {}^{+}2]^{=}$

$[{}^{+}5 \text{ div } 2 = {}^{+}2]^{=}$

$[{}^{+}6 \text{ div } 2 = {}^{+}3]^{=}$

$[{}^{+}7 \text{ div } 2 = {}^{+}3]^{=}$

$[{}^{+}8 \text{ div } 2 = {}^{+}4]^{=}$

$[{}^{+}9 \text{ div } 2 = {}^{+}4]^{=}$

$[{}^{-}9 \text{ div } 3 = {}^{-}3]^{=}$

$[{}^{-}8 \text{ div } 3 = {}^{-}3]^{=}$

$[{}^{-}7 \text{ div } 3 = {}^{-}3]^{=}$

$[{}^{-}6 \text{ div } 3 = {}^{-}2]^{=}$

$[{}^{-}5 \text{ div } 3 = {}^{-}2]^{=}$

$[{}^{-}4 \text{ div } 3 = {}^{-}2]^{=}$

$[{}^{-}3 \text{ div } 3 = {}^{-}1]^{=}$

$[{}^{-}2 \text{ div } 3 = {}^{-}1]^{=}$

$[{}^{-}1 \text{ div } 3 = {}^{-}1]^{=}$

$[{}^{+}0 \text{ div } 3 = {}^{+}0]^{=}$

$[{}^{+}1 \text{ div } 3 = {}^{+}0]^{=}$

$[{}^{+}2 \text{ div } 3 = {}^{+}0]^{=}$

$[{}^{+}3 \text{ div } 3 = {}^{+}1]^{=}$

$[{}^{+}4 \text{ div } 3 = {}^{+}1]^{=}$

$[{}^{+}5 \text{ div } 3 = {}^{+}1]^{=}$

$[{}^{+}6 \text{ div } 3 = {}^{+}2]^{=}$

$[{}^{+}7 \text{ div } 3 = {}^{+}2]^{=}$

$[{}^{+}8 \text{ div } 3 = {}^{+}2]^{=}$

$[{}^{+}9 \text{ div } 3 = {}^{+}3]^{=}$

$[{}^{-}9 \text{ div } 4 = {}^{-}3]^{=}$

$[{}^{-}8 \text{ div } 4 = {}^{-}2]^{=}$

$[{}^{-}7 \text{ div } 4 = {}^{-}2]^{=}$

$[{}^{-}6 \text{ div } 4 = {}^{-}2]^{=}$

$[{}^{-}5 \text{ div } 4 = {}^{-}2]^{=}$

$[{}^{-}4 \text{ div } 4 = {}^{-}1]^{=}$

$[{}^{-}3 \text{ div } 4 = {}^{-}1]^{=}$

$[{}^{-}2 \text{ div } 4 = {}^{-}1]^{=}$

$[{}^{-}1 \text{ div } 4 = {}^{-}1]^{=}$

$[{}^{+}0 \text{ div } 4 = {}^{+}0]^{=}$

$[{}^{+}1 \text{ div } 4 = {}^{+}0]^{=}$

$[{}^{+}2 \text{ div } 4 = {}^{+}0]^{=}$

$[{}^{+}3 \text{ div } 4 = {}^{+}0]^{=}$

$[{}^{+}4 \text{ div } 4 = {}^{+}1]^{=}$

$[{}^{+}5 \text{ div } 4 = {}^{+}1]^{=}$

$[{}^{+}6 \text{ div } 4 = {}^{+}1]^{=}$

$[{}^{+}7 \text{ div } 4 = {}^{+}1]^{=}$

$[{}^{+}8 \text{ div } 4 = {}^{+}2]^{=}$

$[{}^{+}9 \text{ div } 4 = {}^{+}2]^{=}$

$[{}^{-}9 \text{ div } 5 = {}^{-}2]^{=}$

$[{}^{-}8 \text{ div } 5 = {}^{-}2]^{=}$

$[{}^{-}7 \text{ div } 5 = {}^{-}2]^{=}$

$[{}^{-}6 \text{ div } 5 = {}^{-}2]^{=}$

$[{}^{-}5 \text{ div } 5 = {}^{-}1]^{=}$

$[{}^{-}4 \text{ div } 5 = {}^{-}1]^{=}$

$[{}^{-}3 \text{ div } 5 = {}^{-}1]^{=}$

$[{}^{-}2 \text{ div } 5 = {}^{-}1]^{=}$

$[{}^{-}1 \text{ div } 5 = {}^{-}1]^{=}$

$[{}^{+}0 \text{ div } 5 = {}^{+}0]^{=}$

$[{}^{+}1 \text{ div } 5 = {}^{+}0]^{=}$

$[{}^{+}2 \text{ div } 5 = {}^{+}0]^{=}$

$[{}^{+}3 \text{ div } 5 = {}^{+}0]^{=}$

$[{}^{+}4 \text{ div } 5 = {}^{+}0]^{=}$

$[{}^{+}5 \text{ div } 5 = {}^{+}1]^{=}$

$[{}^{+}6 \text{ div } 5 = {}^{+}1]^{=}$

$[{}^{+}7 \text{ div } 5 = {}^{+}1]^{=}$

$[{}^{+}8 \text{ div } 5 = {}^{+}1]^{=}$

$[{}^{+}9 \text{ div } 5 = {}^{+}1]^{=}$

$[{}^{-}5 \text{ div } 0 = \bullet]^{=}$

$[{}^{-}1 \text{ div } 0 = \bullet]^{=}$

$[{}^{+}0 \text{ div } 0 = \bullet]^{=}$

$[{}^{+}1 \text{ div } 0 = \bullet]^{=}$

$[{}^{+}5 \text{ div } 0 = \bullet]^{=}$

$[{}^{-}5 \text{ div } {}^{-}1 = \bullet]^{=}$

$[{}^{-}1 \text{ div } {}^{-}1 = \bullet]^{=}$

$[{}^{+}0 \text{ div } {}^{-}1 = \bullet]^{=}$

$[{}^{+}1 \text{ div } {}^{-}1 = \bullet]^{=}$

$[{}^{+}5 \text{ div } {}^{-}1 = \bullet]^{=}$

13.3 mod

$[{}^{-}9 \text{ mod } 1 = {}^{+}0]^{=}$

$[{}^{-}8 \text{ mod } 1 = {}^{+}0]^{=}$

$[{}^{-}7 \text{ mod } 1 = {}^{+}0]^{=}$

$[{}^{-}6 \text{ mod } 1 = {}^{+}0]^{=}$

$[{}^{-}5 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{-}4 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{-}3 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{-}2 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{-}1 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}0 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}1 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}2 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}3 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}4 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}5 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}6 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}7 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}8 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{+}9 \text{ mod } 1 = {}^{+}0]^{=}$

$[^{-}9 \text{ mod } 2 = {}^{+}1]^{=}$

$[^{-}8 \text{ mod } 2 = {}^{+}0]^{=}$

$[^{-}7 \text{ mod } 2 = {}^{+}1]^{=}$

$[^{-}6 \text{ mod } 2 = {}^{+}0]^{=}$

$[^{-}5 \text{ mod } 2 = {}^{+}1]^{=}$

$[^{-}4 \text{ mod } 2 = {}^{+}0]^{=}$

$[^{-}3 \text{ mod } 2 = {}^{+}1]^{=}$

$[^{-}2 \text{ mod } 2 = {}^{+}0]^{=}$

$[^{-}1 \text{ mod } 2 = {}^{+}1]^{=}$

$[^{+}0 \text{ mod } 2 = {}^{+}0]^{=}$

$[^{+}1 \text{ mod } 2 = {}^{+}1]^{=}$

$[^{+}2 \text{ mod } 2 = {}^{+}0]^{=}$

$[^{+}3 \text{ mod } 2 = {}^{+}1]^{=}$

$[^{+}4 \text{ mod } 2 = {}^{+}0]^{=}$

$[^{+}5 \bmod 2 = {}^{+}1]^{=}$

$[^{+}6 \bmod 2 = {}^{+}0]^{=}$

$[^{+}7 \bmod 2 = {}^{+}1]^{=}$

$[^{+}8 \bmod 2 = {}^{+}0]^{=}$

$[^{+}9 \bmod 2 = {}^{+}1]^{=}$

$[^{-}9 \bmod 3 = {}^{+}0]^{=}$

$[^{-}8 \bmod 3 = {}^{+}1]^{=}$

$[^{-}7 \bmod 3 = {}^{+}2]^{=}$

$[^{-}6 \bmod 3 = {}^{+}0]^{=}$

$[^{-}5 \bmod 3 = {}^{+}1]^{=}$

$[^{-}4 \bmod 3 = {}^{+}2]^{=}$

$[^{-}3 \bmod 3 = {}^{+}0]^{=}$

$[^{-}2 \bmod 3 = {}^{+}1]^{=}$

$[^{-}1 \bmod 3 = {}^{+}2]^{=}$

$[^{+}0 \bmod 3 = {}^{+}0]^{=}$

$[^{+}1 \bmod 3 = {}^{+}1]^{=}$

$[^{+}2 \bmod 3 = {}^{+}2]^{=}$

$[^{+}3 \bmod 3 = {}^{+}0]^{=}$

$[^{+}4 \bmod 3 = {}^{+}1]^{=}$

$[^{+}5 \bmod 3 = {}^{+}2]^{=}$

$[^{+}6 \bmod 3 = {}^{+}0]^{=}$

$[^{+}7 \bmod 3 = {}^{+}1]^{=}$

$[^{+}8 \bmod 3 = {}^{+}2]^{=}$

$[^{+}9 \bmod 3 = {}^{+}0]^{=}$

$[^{-}9 \bmod 4 = {}^{+}3]^{=}$

$[^{-}8 \bmod 4 = {}^{+}0]^{=}$

$[^{-}7 \bmod 4 = {}^{+}1]^{=}$

$[^{-}6 \bmod 4 = {}^{+}2]^{=}$

$[^{-}5 \bmod 4 = {}^{+}3]^{=}$

$[^{-}4 \bmod 4 = {}^{+}0]^{=}$

$[^{-}3 \bmod 4 = {}^{+}1]^{=}$

$[^{-}2 \bmod 4 = {}^{+}2]^{=}$

$[^{-}1 \bmod 4 = {}^{+}3]^{=}$

$[^{+}0 \bmod 4 = {}^{+}0]^{=}$

$[^{+}1 \bmod 4 = {}^{+}1]^{=}$

$[^{+}2 \bmod 4 = {}^{+}2]^{=}$

$[^{+}3 \bmod 4 = {}^{+}3]^{=}$

$[^{+}4 \bmod 4 = {}^{+}0]^{=}$

$[^{+}5 \bmod 4 = {}^{+}1]^{=}$

$[^{+}6 \bmod 4 = {}^{+}2]^{=}$

$[^{+}7 \bmod 4 = {}^{+}3]^{=}$

$[^{+}8 \bmod 4 = {}^{+}0]^{=}$

$[^{+}9 \bmod 4 = {}^{+}1]^{=}$

$[^{-}9 \bmod 5 = {}^{+}1]^{=}$

$[^{-}8 \bmod 5 = {}^{+}2]^{=}$

$[^{-}7 \bmod 5 = {}^{+}3]^{=}$

$[^{-}6 \bmod 5 = {}^{+}4]^{=}$

$[^{-}5 \bmod 5 = {}^{+}0]^{=}$

$[^{-}4 \bmod 5 = {}^{+}1]^{=}$

$[^{-}3 \bmod 5 = {}^{+}2]^{=}$

$[^{-}2 \bmod 5 = {}^{+}3]^{=}$

$[^{-}1 \bmod 5 = {}^{+}4]^{=}$

$[^{+}0 \bmod 5 = {}^{+}0]^{=}$

$[^{+}1 \bmod 5 = {}^{+}1]^{=}$

$[^{+}2 \bmod 5 = {}^{+}2]^{=}$

$[^{+}3 \bmod 5 = {}^{+}3]^{=}$

$[^{+}4 \bmod 5 = {}^{+}4]^{=}$

$[^{+}5 \bmod 5 = {}^{+}0]^{=}$

$[^{+}6 \bmod 5 = {}^{+}1]^{=}$

$[^{+}7 \bmod 5 = {}^{+}2]^{=}$

$[^{+}8 \bmod 5 = {}^{+}3]^{=}$

$[^{+}9 \bmod 5 = {}^{+}4]^{=}$

$[^{-}5 \bmod 0 = \bullet]^{=}$

$[^{-}1 \bmod 0 = \bullet]^{=}$

$[^{+}0 \bmod 0 = \bullet]^{=}$

$[^{+}1 \bmod 0 = \bullet]^{=}$

$[^{+}5 \bmod 0 = \bullet]^{=}$

$[^{-}5 \bmod {}^{-}1 = \bullet]^{=}$

$[^{-}1 \bmod {}^{-}1 = \bullet]^{=}$

$[^{+}0 \bmod {}^{-}1 = \bullet]^{=}$

$[^{+}1 \bmod {}^{-}1 = \bullet]^{=}$

$[^{+}5 \bmod {}^{-}1 = \bullet]^{=}$

13.4 ceiling

$[\text{ceiling } (\ {}^{+}9 \ , \ 1 \ ) = {}^{+}9 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}8 \ , \ 1 \ ) = {}^{+}8 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}7 \ , \ 1 \ ) = {}^{+}7 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}6 \ , \ 1 \ ) = {}^{+}6 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}5 \ , \ 1 \ ) = {}^{+}5 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}4 \ , \ 1 \ ) = {}^{+}4 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}3 \ , \ 1 \ ) = {}^{+}3 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}2 \ , \ 1 \ ) = {}^{+}2 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{+}1 \ , \ 1 \ ) = {}^{+}1 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{-}0 \ , \ 1 \ ) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{ceiling } (\ {}^{-}1 \ , \ 1 \ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}2\ ,\ 1\ ) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}3\ ,\ 1\ ) = {}^{-}3 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}4\ ,\ 1\ ) = {}^{-}4 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}5\ ,\ 1\ ) = {}^{-}5 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}6\ ,\ 1\ ) = {}^{-}6 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}7\ ,\ 1\ ) = {}^{-}7 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}8\ ,\ 1\ ) = {}^{-}8 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}9\ ,\ 1\ ) = {}^{-}9 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{+}9\ ,\ 2\ ) = {}^{+}5 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}8\ ,\ 2\ ) = {}^{+}4 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{+}7\ ,\ 2\ ) = {}^{+}4 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}6\ ,\ 2\ ) = {}^{+}3 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{+}5\ ,\ 2\ ) = {}^{+}3 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}4\ ,\ 2\ ) = {}^{+}2 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{+}3\ ,\ 2\ ) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}2\ ,\ 2\ ) = {}^{+}1 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{+}1\ ,\ 2\ ) = {}^{+}1 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}0\ ,\ 2\ ) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}1\ ,\ 2\ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}2\ ,\ 2\ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}3\ ,\ 2\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}4\ ,\ 2\ ) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}5\ ,\ 2\ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}6\ ,\ 2\ ) = {}^{-}3 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}7\ ,\ 2\ ) = {}^{-}3 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}8\ ,\ 2\ ) = {}^{-}4 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}9\ ,\ 2\ ) = {}^{-}4 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}9\ ,\ 3\ ) = {}^{+}3 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{+}8 , 3) = {}^{+}3 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{+}7 , 3) = {}^{+}3 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{+}6 , 3) = {}^{+}2 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{+}5 , 3) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{+}4 , 3) = {}^{+}2 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{+}3 , 3) = {}^{+}1 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{+}2 , 3) = {}^{+}1 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{+}1 , 3) = {}^{+}1 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{-}0 , 3) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{-}1 , 3) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{-}2 , 3) = {}^{-}0 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{-}3 , 3) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{-}4 , 3) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{-}5 , 3) = {}^{-}1 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{-}6 , 3) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{-}7 , 3) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{-}8 , 3) = {}^{-}2 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{-}9 , 3) = {}^{-}3 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{+}9 , 4) = {}^{+}3 :: {}^{-}3]^{=}$

$[\text{ceiling} ({}^{+}8 , 4) = {}^{+}2 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{+}7 , 4) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{+}6 , 4) = {}^{+}2 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{+}5 , 4) = {}^{+}2 :: {}^{-}3]^{=}$

$[\text{ceiling} ({}^{+}4 , 4) = {}^{+}1 :: {}^{-}0]^{=}$

$[\text{ceiling} ({}^{+}3 , 4) = {}^{+}1 :: {}^{-}1]^{=}$

$[\text{ceiling} ({}^{+}2 , 4) = {}^{+}1 :: {}^{-}2]^{=}$

$[\text{ceiling} ({}^{+}1 , 4) = {}^{+}1 :: {}^{-}3]^{=}$

$[\text{ceiling} ({}^{-}0 , 4) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}1\ ,\ 4\ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}2\ ,\ 4\ ) = {}^{-}0 :: {}^{-}2]^{=}$

$[\text{ceiling}\ (\ {}^{-}3\ ,\ 4\ ) = {}^{-}0 :: {}^{-}3]^{=}$

$[\text{ceiling}\ (\ {}^{-}4\ ,\ 4\ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}5\ ,\ 4\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}6\ ,\ 4\ ) = {}^{-}1 :: {}^{-}2]^{=}$

$[\text{ceiling}\ (\ {}^{-}7\ ,\ 4\ ) = {}^{-}1 :: {}^{-}3]^{=}$

$[\text{ceiling}\ (\ {}^{-}8\ ,\ 4\ ) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}9\ ,\ 4\ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}9\ ,\ 5\ ) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}8\ ,\ 5\ ) = {}^{+}2 :: {}^{-}2]^{=}$

$[\text{ceiling}\ (\ {}^{+}7\ ,\ 5\ ) = {}^{+}2 :: {}^{-}3]^{=}$

$[\text{ceiling}\ (\ {}^{+}6\ ,\ 5\ ) = {}^{+}2 :: {}^{-}4]^{=}$

$[\text{ceiling}\ (\ {}^{+}5\ ,\ 5\ ) = {}^{+}1 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{+}4\ ,\ 5\ ) = {}^{+}1 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{+}3\ ,\ 5\ ) = {}^{+}1 :: {}^{-}2]^{=}$

$[\text{ceiling}\ (\ {}^{+}2\ ,\ 5\ ) = {}^{+}1 :: {}^{-}3]^{=}$

$[\text{ceiling}\ (\ {}^{+}1\ ,\ 5\ ) = {}^{+}1 :: {}^{-}4]^{=}$

$[\text{ceiling}\ (\ {}^{-}0\ ,\ 5\ ) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}1\ ,\ 5\ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}2\ ,\ 5\ ) = {}^{-}0 :: {}^{-}2]^{=}$

$[\text{ceiling}\ (\ {}^{-}3\ ,\ 5\ ) = {}^{-}0 :: {}^{-}3]^{=}$

$[\text{ceiling}\ (\ {}^{-}4\ ,\ 5\ ) = {}^{-}0 :: {}^{-}4]^{=}$

$[\text{ceiling}\ (\ {}^{-}5\ ,\ 5\ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{ceiling}\ (\ {}^{-}6\ ,\ 5\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{ceiling}\ (\ {}^{-}7\ ,\ 5\ ) = {}^{-}1 :: {}^{-}2]^{=}$

$[\text{ceiling}\ (\ {}^{-}8\ ,\ 5\ ) = {}^{-}1 :: {}^{-}3]^{=}$

$[\text{ceiling}\ (\ {}^{-}9\ ,\ 5\ ) = {}^{-}1 :: {}^{-}4]^{=}$

$[\text{ceiling} ({}^{+}5 , 0) = \bullet]^{=}$

$[\text{ceiling} ({}^{+}1 , 0) = \bullet]^{=}$

$[\text{ceiling} ({}^{-}0 , 0) = \bullet]^{=}$

$[\text{ceiling} ({}^{-}1 , 0) = \bullet]^{=}$

$[\text{ceiling} ({}^{-}5 , 0) = \bullet]^{=}$

$[\text{ceiling} ({}^{+}5 , {}^{-}1) = \bullet]^{=}$

$[\text{ceiling} ({}^{+}1 , {}^{-}1) = \bullet]^{=}$

$[\text{ceiling} ({}^{-}0 , {}^{-}1) = \bullet]^{=}$

$[\text{ceiling} ({}^{-}1 , {}^{-}1) = \bullet]^{=}$

$[\text{ceiling} ({}^{-}5 , {}^{-}1) = \bullet]^{=}$

13.5 truncate

$[\text{truncate} ({}^{+}0 , 1) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}1 , 1) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}2 , 1) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}3 , 1) = {}^{+}3 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}4 , 1) = {}^{+}4 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}5 , 1) = {}^{+}5 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}6 , 1) = {}^{+}6 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}7 , 1) = {}^{+}7 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}8 , 1) = {}^{+}8 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{+}9 , 1) = {}^{+}9 :: {}^{+}0]^{=}$

$[\text{truncate} ({}^{-}0 , 1) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{truncate} ({}^{-}1 , 1) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{truncate} ({}^{-}2 , 1) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{truncate} ({}^{-}3 , 1) = {}^{-}3 :: {}^{-}0]^{=}$

$[\text{truncate} ({}^{-}4 , 1) = {}^{-}4 :: {}^{-}0]^{=}$

$[\text{truncate} ({}^{-}5 , 1) = {}^{-}5 :: {}^{-}0]^{=}$

$[\text{truncate} ({}^{-}6 , 1) = {}^{-}6 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}7 \ , \ 1 \ ) = {}^{-}7 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}8 \ , \ 1 \ ) = {}^{-}8 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}9 \ , \ 1 \ ) = {}^{-}9 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{+}0 \ , \ 2 \ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{truncate} \ (\ {}^{+}1 \ , \ 2 \ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{truncate} \ (\ {}^{+}2 \ , \ 2 \ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{truncate} \ (\ {}^{+}3 \ , \ 2 \ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{truncate} \ (\ {}^{+}4 \ , \ 2 \ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{truncate} \ (\ {}^{+}5 \ , \ 2 \ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{truncate} \ (\ {}^{+}6 \ , \ 2 \ ) = {}^{+}3 :: {}^{+}0]^{=}$

$[\text{truncate} \ (\ {}^{+}7 \ , \ 2 \ ) = {}^{+}3 :: {}^{+}1]^{=}$

$[\text{truncate} \ (\ {}^{+}8 \ , \ 2 \ ) = {}^{+}4 :: {}^{+}0]^{=}$

$[\text{truncate} \ (\ {}^{+}9 \ , \ 2 \ ) = {}^{+}4 :: {}^{+}1]^{=}$

$[\text{truncate} \ (\ {}^{-}0 \ , \ 2 \ ) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}1 \ , \ 2 \ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{truncate} \ (\ {}^{-}2 \ , \ 2 \ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}3 \ , \ 2 \ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{truncate} \ (\ {}^{-}4 \ , \ 2 \ ) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}5 \ , \ 2 \ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{truncate} \ (\ {}^{-}6 \ , \ 2 \ ) = {}^{-}3 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}7 \ , \ 2 \ ) = {}^{-}3 :: {}^{-}1]^{=}$

$[\text{truncate} \ (\ {}^{-}8 \ , \ 2 \ ) = {}^{-}4 :: {}^{-}0]^{=}$

$[\text{truncate} \ (\ {}^{-}9 \ , \ 2 \ ) = {}^{-}4 :: {}^{-}1]^{=}$

$[\text{truncate} \ (\ {}^{+}0 \ , \ 3 \ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{truncate} \ (\ {}^{+}1 \ , \ 3 \ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{truncate} \ (\ {}^{+}2 \ , \ 3 \ ) = {}^{+}0 :: {}^{+}2]^{=}$

$[\text{truncate} \ (\ {}^{+}3 \ , \ 3 \ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{truncate} \ (\ {}^{+}4 \ , \ 3 \ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{truncate}\ (\ {}^{+}5\ ,\ 3\ ) = {}^{+}1 :: {}^{+}2]^{=}$

$[\text{truncate}\ (\ {}^{+}6\ ,\ 3\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{truncate}\ (\ {}^{+}7\ ,\ 3\ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{truncate}\ (\ {}^{+}8\ ,\ 3\ ) = {}^{+}2 :: {}^{+}2]^{=}$

$[\text{truncate}\ (\ {}^{+}9\ ,\ 3\ ) = {}^{+}3 :: {}^{+}0]^{=}$

$[\text{truncate}\ (\ {}^{-}0\ ,\ 3\ ) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}1\ ,\ 3\ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{-}2\ ,\ 3\ ) = {}^{-}0 :: {}^{-}2]^{=}$

$[\text{truncate}\ (\ {}^{-}3\ ,\ 3\ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}4\ ,\ 3\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{-}5\ ,\ 3\ ) = {}^{-}1 :: {}^{-}2]^{=}$

$[\text{truncate}\ (\ {}^{-}6\ ,\ 3\ ) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}7\ ,\ 3\ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{-}8\ ,\ 3\ ) = {}^{-}2 :: {}^{-}2]^{=}$

$[\text{truncate}\ (\ {}^{-}9\ ,\ 3\ ) = {}^{-}3 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{+}0\ ,\ 4\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{truncate}\ (\ {}^{+}1\ ,\ 4\ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{truncate}\ (\ {}^{+}2\ ,\ 4\ ) = {}^{+}0 :: {}^{+}2]^{=}$

$[\text{truncate}\ (\ {}^{+}3\ ,\ 4\ ) = {}^{+}0 :: {}^{+}3]^{=}$

$[\text{truncate}\ (\ {}^{+}4\ ,\ 4\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{truncate}\ (\ {}^{+}5\ ,\ 4\ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{truncate}\ (\ {}^{+}6\ ,\ 4\ ) = {}^{+}1 :: {}^{+}2]^{=}$

$[\text{truncate}\ (\ {}^{+}7\ ,\ 4\ ) = {}^{+}1 :: {}^{+}3]^{=}$

$[\text{truncate}\ (\ {}^{+}8\ ,\ 4\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{truncate}\ (\ {}^{+}9\ ,\ 4\ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{truncate}\ (\ {}^{-}0\ ,\ 4\ ) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}1\ ,\ 4\ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{-}2\ ,\ 4\ ) = {}^{-}0 :: {}^{-}2]^{=}$

$[\text{truncate}\ (\ {}^{-}3\ ,\ 4\ ) = {}^{-}0 :: {}^{-}3]^{=}$

$[\text{truncate}\ (\ {}^{-}4\ ,\ 4\ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}5\ ,\ 4\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{-}6\ ,\ 4\ ) = {}^{-}1 :: {}^{-}2]^{=}$

$[\text{truncate}\ (\ {}^{-}7\ ,\ 4\ ) = {}^{-}1 :: {}^{-}3]^{=}$

$[\text{truncate}\ (\ {}^{-}8\ ,\ 4\ ) = {}^{-}2 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}9\ ,\ 4\ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{+}0\ ,\ 5\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{truncate}\ (\ {}^{+}1\ ,\ 5\ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{truncate}\ (\ {}^{+}2\ ,\ 5\ ) = {}^{+}0 :: {}^{+}2]^{=}$

$[\text{truncate}\ (\ {}^{+}3\ ,\ 5\ ) = {}^{+}0 :: {}^{+}3]^{=}$

$[\text{truncate}\ (\ {}^{+}4\ ,\ 5\ ) = {}^{+}0 :: {}^{+}4]^{=}$

$[\text{truncate}\ (\ {}^{+}5\ ,\ 5\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{truncate}\ (\ {}^{+}6\ ,\ 5\ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{truncate}\ (\ {}^{+}7\ ,\ 5\ ) = {}^{+}1 :: {}^{+}2]^{=}$

$[\text{truncate}\ (\ {}^{+}8\ ,\ 5\ ) = {}^{+}1 :: {}^{+}3]^{=}$

$[\text{truncate}\ (\ {}^{+}9\ ,\ 5\ ) = {}^{+}1 :: {}^{+}4]^{=}$

$[\text{truncate}\ (\ {}^{-}0\ ,\ 5\ ) = {}^{-}0 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}1\ ,\ 5\ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{-}2\ ,\ 5\ ) = {}^{-}0 :: {}^{-}2]^{=}$

$[\text{truncate}\ (\ {}^{-}3\ ,\ 5\ ) = {}^{-}0 :: {}^{-}3]^{=}$

$[\text{truncate}\ (\ {}^{-}4\ ,\ 5\ ) = {}^{-}0 :: {}^{-}4]^{=}$

$[\text{truncate}\ (\ {}^{-}5\ ,\ 5\ ) = {}^{-}1 :: {}^{-}0]^{=}$

$[\text{truncate}\ (\ {}^{-}6\ ,\ 5\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{truncate}\ (\ {}^{-}7\ ,\ 5\ ) = {}^{-}1 :: {}^{-}2]^{=}$

$[\text{truncate}\ (\ {}^{-}8\ ,\ 5\ ) = {}^{-}1 :: {}^{-}3]^{=}$

$[\text{truncate}\ (\ {}^{-}9\ ,\ 5\ ) = {}^{-}1 :: {}^{-}4]^{=}$

$[\text{truncate}\ (\ {}^{-}5\ ,\ 0\ ) = \bullet]^{=}$

[truncate ($^{-}1$, 0) = •]$^{=}$

[truncate ($^{+}0$, 0) = •]$^{=}$

[truncate ($^{+}1$, 0) = •]$^{=}$

[truncate ($^{+}5$, 0) = •]$^{=}$

[truncate ($^{-}5$, $^{-}1$) = •]$^{=}$

[truncate ($^{-}1$, $^{-}1$) = •]$^{=}$

[truncate ($^{+}0$, $^{-}1$) = •]$^{=}$

[truncate ($^{+}1$, $^{-}1$) = •]$^{=}$

[truncate ($^{+}5$, $^{-}1$) = •]$^{=}$

13.6 round

[round ($^{-}9$, 1) = $^{-}9$: : $^{+}0$]$^{=}$

[round ($^{-}8$, 1) = $^{-}8$: : $^{+}0$]$^{=}$

[round ($^{-}7$, 1) = $^{-}7$: : $^{+}0$]$^{=}$

[round ($^{-}6$, 1) = $^{-}6$: : $^{+}0$]$^{=}$

[round ($^{-}5$, 1) = $^{-}5$: : $^{+}0$]$^{=}$

[round ($^{-}4$, 1) = $^{-}4$: : $^{+}0$]$^{=}$

[round ($^{-}3$, 1) = $^{-}3$: : $^{+}0$]$^{=}$

[round ($^{-}2$, 1) = $^{-}2$: : $^{+}0$]$^{=}$

[round ($^{-}1$, 1) = $^{-}1$: : $^{+}0$]$^{=}$

[round ($^{+}0$, 1) = $^{+}0$: : $^{+}0$]$^{=}$

[round ($^{+}1$, 1) = $^{+}1$: : $^{+}0$]$^{=}$

[round ($^{+}2$, 1) = $^{+}2$: : $^{+}0$]$^{=}$

[round ($^{+}3$, 1) = $^{+}3$: : $^{+}0$]$^{=}$

[round ($^{+}4$, 1) = $^{+}4$: : $^{+}0$]$^{=}$

[round ($^{+}5$, 1) = $^{+}5$: : $^{+}0$]$^{=}$

[round ($^{+}6$, 1) = $^{+}6$: : $^{+}0$]$^{=}$

[round ($^{+}7$, 1) = $^{+}7$: : $^{+}0$]$^{=}$

[round ($^{+}8$, 1) = $^{+}8$: : $^{+}0$]$^{=}$

$[\text{round}\ (\ {}^{+}9\ ,\ 1\ ) = {}^{+}9 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}9\ ,\ 2\ ) = {}^{-}4 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{-}8\ ,\ 2\ ) = {}^{-}4 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}7\ ,\ 2\ ) = {}^{-}4 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}6\ ,\ 2\ ) = {}^{-}3 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}5\ ,\ 2\ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{-}4\ ,\ 2\ ) = {}^{-}2 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}3\ ,\ 2\ ) = {}^{-}2 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}2\ ,\ 2\ ) = {}^{-}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}1\ ,\ 2\ ) = {}^{-}0 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{+}0\ ,\ 2\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}1\ ,\ 2\ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{+}2\ ,\ 2\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}3\ ,\ 2\ ) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{+}4\ ,\ 2\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}5\ ,\ 2\ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{+}6\ ,\ 2\ ) = {}^{+}3 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}7\ ,\ 2\ ) = {}^{+}4 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{+}8\ ,\ 2\ ) = {}^{+}4 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}9\ ,\ 2\ ) = {}^{+}4 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}9\ ,\ 3\ ) = {}^{-}3 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}8\ ,\ 3\ ) = {}^{-}3 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}7\ ,\ 3\ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{-}6\ ,\ 3\ ) = {}^{-}2 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}5\ ,\ 3\ ) = {}^{-}2 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}4\ ,\ 3\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{-}3\ ,\ 3\ ) = {}^{-}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}2\ ,\ 3\ ) = {}^{-}1 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{-}1\ ,\ 3\ ) = {}^{+}0 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{+}0\ ,\ 3\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{+}1\ ,\ 3\ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{+}2\ ,\ 3\ ) = {}^{+}1 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{+}3\ ,\ 3\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{+}4\ ,\ 3\ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{+}5\ ,\ 3\ ) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{+}6\ ,\ 3\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{+}7\ ,\ 3\ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{+}8\ ,\ 3\ ) = {}^{+}3 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{+}9\ ,\ 3\ ) = {}^{+}3 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{-}9\ ,\ 4\ ) = {}^{-}2 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{-}8\ ,\ 4\ ) = {}^{-}2 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{-}7\ ,\ 4\ ) = {}^{-}2 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{-}6\ ,\ 4\ ) = {}^{-}2 :: {}^{+}2]^{=}$

$[\text{round}\ (\ ^{-}5\ ,\ 4\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{-}4\ ,\ 4\ ) = {}^{-}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{-}3\ ,\ 4\ ) = {}^{-}1 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{-}2\ ,\ 4\ ) = {}^{+}0 :: {}^{-}2]^{=}$

$[\text{round}\ (\ ^{-}1\ ,\ 4\ ) = {}^{+}0 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{+}0\ ,\ 4\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{+}1\ ,\ 4\ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{+}2\ ,\ 4\ ) = {}^{+}0 :: {}^{+}2]^{=}$

$[\text{round}\ (\ ^{+}3\ ,\ 4\ ) = {}^{+}1 :: {}^{-}1]^{=}$

$[\text{round}\ (\ ^{+}4\ ,\ 4\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ ^{+}5\ ,\ 4\ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{round}\ (\ ^{+}6\ ,\ 4\ ) = {}^{+}2 :: {}^{-}2]^{=}$

$[\text{round}\ (\ ^{+}7\ ,\ 4\ ) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{+}8\ ,\ 4\ ) = {}^{+}2 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}9\ ,\ 4\ ) = {}^{+}2 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}9\ ,\ 5\ ) = {}^{-}2 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}8\ ,\ 5\ ) = {}^{-}2 :: {}^{+}2]^{=}$

$[\text{round}\ (\ {}^{-}7\ ,\ 5\ ) = {}^{-}1 :: {}^{-}2]^{=}$

$[\text{round}\ (\ {}^{-}6\ ,\ 5\ ) = {}^{-}1 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{-}5\ ,\ 5\ ) = {}^{-}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{-}4\ ,\ 5\ ) = {}^{-}1 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{-}3\ ,\ 5\ ) = {}^{-}1 :: {}^{+}2]^{=}$

$[\text{round}\ (\ {}^{-}2\ ,\ 5\ ) = {}^{+}0 :: {}^{-}2]^{=}$

$[\text{round}\ (\ {}^{-}1\ ,\ 5\ ) = {}^{+}0 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{+}0\ ,\ 5\ ) = {}^{+}0 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}1\ ,\ 5\ ) = {}^{+}0 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{+}2\ ,\ 5\ ) = {}^{+}0 :: {}^{+}2]^{=}$

$[\text{round}\ (\ {}^{+}3\ ,\ 5\ ) = {}^{+}1 :: {}^{-}2]^{=}$

$[\text{round}\ (\ {}^{+}4\ ,\ 5\ ) = {}^{+}1 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{+}5\ ,\ 5\ ) = {}^{+}1 :: {}^{+}0]^{=}$

$[\text{round}\ (\ {}^{+}6\ ,\ 5\ ) = {}^{+}1 :: {}^{+}1]^{=}$

$[\text{round}\ (\ {}^{+}7\ ,\ 5\ ) = {}^{+}1 :: {}^{+}2]^{=}$

$[\text{round}\ (\ {}^{+}8\ ,\ 5\ ) = {}^{+}2 :: {}^{-}2]^{=}$

$[\text{round}\ (\ {}^{+}9\ ,\ 5\ ) = {}^{+}2 :: {}^{-}1]^{=}$

$[\text{round}\ (\ {}^{-}5\ ,\ 0\ ) = \bullet]^{=}$

$[\text{round}\ (\ {}^{-}1\ ,\ 0\ ) = \bullet]^{=}$

$[\text{round}\ (\ {}^{+}0\ ,\ 0\ ) = \bullet]^{=}$

$[\text{round}\ (\ {}^{+}1\ ,\ 0\ ) = \bullet]^{=}$

$[\text{round}\ (\ {}^{+}5\ ,\ 0\ ) = \bullet]^{=}$

$[\text{round}\ (\ {}^{-}5\ ,\ {}^{-}1\ ) = \bullet]^{=}$

$[\text{round}\ (\ {}^{-}1\ ,\ {}^{-}1\ ) = \bullet]^{=}$

$[\text{round} \ (\ {}^{+}0 \ , \ {}^{-}1 \ ) = \bullet]^{=}$

$[\text{round} \ (\ {}^{+}1 \ , \ {}^{-}1 \ ) = \bullet]^{=}$

$[\text{round} \ (\ {}^{+}5 \ , \ {}^{-}1 \ ) = \bullet]^{=}$

14 Vectors

14.1 Elementary operations

$[\text{vector-empty} \ (\quad)]^{\cdot}$

$[\text{vector-empty} \ (\ \text{a} \ )]^{-}$

$[\text{vector-empty} \ (\ {}^{-}1 \ )]^{\cdot}$

$[\text{vector-empty} \ (\ 255 \ )]^{\cdot}$

$[\text{vector-head1} \ (\ \text{`abc'} \ ) = 97]^{=}$

$[\text{vector-tail1} \ (\ \text{`abc'} \ ) = \text{`bc'}]^{=}$

14.2 vector

$[\text{vector} \ (\ \text{`abc'} \ ) = \text{`abc'}]^{=}$

$[\text{vector} \ (\ 0 \ ) = \text{`'}]^{=}$

$[\text{vector} \ (\ 1 \ ) = \text{`'}]^{=}$

$[\text{vector} \ (\ 97 \ ) = \text{`'}]^{=}$

$[\text{vector} \ (\ 97 + 256 \cdot 1 \ ) = \text{`a'}]^{=}$

$[\text{vector} \ (\ 97 + 256 \cdot 2 \ ) = \text{`a'}]^{=}$

$[\text{vector} \ (\ 97 + 256 \cdot 255 \ ) = \text{`a'}]^{=}$

$[\text{vector} \ (\ 97 + 256 \cdot 256 \ ) = 97 + 256 \cdot 256]^{=}$

$[\text{vector} \ (\ {}^{-}1 \ ) = \text{`'}]^{=}$

$[\text{vector} \ (\ {}^{-}1000 \ ) = \text{`'}]^{=}$

14.3 vector-norm

$[\text{vector-norm}\ (\ \text{`abc'}\ ) = \text{`abc'}]^{=}$

$[\text{vector-norm}\ (\ 0\ ) = \mathsf{T}]^{=}$

$[\text{vector-norm}\ (\ 1\ ) = \text{`'}]^{=}$

$[\text{vector-norm}\ (\ 97\ ) = \mathsf{T}]^{=}$

$[\text{vector-norm}\ (\ 97 + 256 \cdot 1\ ) = \text{`a'}]^{=}$

$[\text{vector-norm}\ (\ 97 + 256 \cdot 2\ ) = \mathsf{T}]^{=}$

$[\text{vector-norm}\ (\ 97 + 256 \cdot 255\ ) = \mathsf{T}]^{=}$

$[\text{vector-norm}\ (\ 97 + 256 \cdot 256\ ) = 97 + 256 \cdot 256]^{=}$

$[\text{vector-norm}\ (\ {}^{-}1\ ) = \mathsf{T}]^{=}$

$[\text{vector-norm}\ (\ {}^{-}1000\ ) = \mathsf{T}]^{=}$

14.4 vector-prefix

$[\text{vector-prefix}\ (\ \text{`abc'}\ ,\ {}^{-}1\ ) = \text{`'}]^{=}$

$[\text{vector-prefix}\ (\ \text{`abc'}\ ,\ 0\ ) = \text{`'}]^{=}$

$[\text{vector-prefix}\ (\ \text{`abc'}\ ,\ 1\ ) = \text{`a'}]^{=}$

$[\text{vector-prefix}\ (\ \text{`abc'}\ ,\ 2\ ) = \text{`ab'}]^{=}$

$[\text{vector-prefix}\ (\ \text{`abc'}\ ,\ 3\ ) = \text{`abc'}]^{=}$

$[\text{vector-prefix}\ (\ \text{`abc'}\ ,\ 4\ ) = \text{`abc'}]^{=}$

$[\text{vector-prefix}\ (\ 97 + 98 \cdot 256 + 2 \cdot 65536\ ,\ {}^{-}1\ ) = \text{`'}]^{=}$

$[\text{vector-prefix}\ (\ 97 + 98 \cdot 256 + 2 \cdot 65536\ ,\ 0\ ) = \text{`'}]^{=}$

$[\text{vector-prefix}\ (\ 97 + 98 \cdot 256 + 2 \cdot 65536\ ,\ 1\ ) = \text{`a'}]^{=}$

$[\text{vector-prefix}\ (\ 97 + 98 \cdot 256 + 2 \cdot 65536\ ,\ 2\ ) = \text{`ab'}]^{=}$

$[\text{vector-prefix}\ (\ 97 + 98 \cdot 256 + 2 \cdot 65536\ ,\ 3\ ) = \text{`ab'}]^{=}$

$[\text{vector-prefix}\ (\ {}^{-}1000\ ,\ 3\ ) = \text{`'}]^{=}$

14.5 vector-suffix

[vector-suffix (‘abc’ , ¯1) = ‘abc’]$^{=}$

[vector-suffix (‘abc’ , 0) = ‘abc’]$^{=}$

[vector-suffix (‘abc’ , 1) = ‘bc’]$^{=}$

[vector-suffix (‘abc’ , 2) = ‘c’]$^{=}$

[vector-suffix (‘abc’ , 3) = ‘’]$^{=}$

[vector-suffix (‘abc’ , 4) = ‘’]$^{=}$

[vector-suffix ($97 + 98 \cdot 256 + 2 \cdot 65536$, ¯1) = ‘ab’]$^{=}$

[vector-suffix ($97 + 98 \cdot 256 + 2 \cdot 65536$, 0) = ‘ab’]$^{=}$

[vector-suffix ($97 + 98 \cdot 256 + 2 \cdot 65536$, 1) = ‘b’]$^{=}$

[vector-suffix ($97 + 98 \cdot 256 + 2 \cdot 65536$, 2) = ‘’]$^{=}$

[vector-suffix ($97 + 98 \cdot 256 + 2 \cdot 65536$, 3) = ‘’]$^{=}$

[vector-suffix (¯1000 , 0) = ‘’]$^{=}$

14.6 vector-subseq

[vector-subseq (‘ab’ , ¯1 , ¯1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , ¯1 , 0) = ‘’]$^{=}$

[vector-subseq (‘ab’ , ¯1 , 1) = ‘a’]$^{=}$

[vector-subseq (‘ab’ , ¯1 , 2) = ‘ab’]$^{=}$

[vector-subseq (‘ab’ , ¯1 , 3) = ‘ab’]$^{=}$

[vector-subseq (‘ab’ , 0 , ¯1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 0 , 0) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 0 , 1) = ‘a’]$^{=}$

[vector-subseq (‘ab’ , 0 , 2) = ‘ab’]$^{=}$

[vector-subseq (‘ab’ , 0 , 3) = ‘ab’]$^{=}$

[vector-subseq (‘ab’ , 1 , ¯1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 1 , 0) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 1 , 1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 1 , 2) = ‘b’]$^{=}$

[vector-subseq (‘ab’ , 1 , 3) = ‘b’]$^{=}$

[vector-subseq (‘ab’ , 2 , ¯1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 2 , 0) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 2 , 1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 2 , 2) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 2 , 3) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 3 , ¯1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 3 , 0) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 3 , 1) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 3 , 2) = ‘’]$^{=}$

[vector-subseq (‘ab’ , 3 , 3) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, ¯1 , ¯1) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, ¯1 , 0) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, ¯1 , 1) = ‘a’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, ¯1 , 2) = ‘ab’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, ¯1 , 3) = ‘ab’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 0 , ¯1) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 0 , 0) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 0 , 1) = ‘a’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 0 , 2) = ‘ab’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 0 , 3) = ‘ab’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 1 , ¯1) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 1 , 0) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 1 , 1) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 1 , 2) = ‘b’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 1 , 3) = ‘b’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 2 , ¯1) = ‘’]$^{=}$

[vector-subseq ($97 + 98 \cdot 256 + 2 \cdot 65536$, 2 , 0) = ‘’]$^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 2 \ , \ 1) = \text{‘’}]^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 2 \ , \ 2) = \text{‘’}]^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 2 \ , \ 3) = \text{‘’}]^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 3 \ , \ {}^{-}1) = \text{‘’}]^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 3 \ , \ 0) = \text{‘’}]^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 3 \ , \ 1) = \text{‘’}]^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 3 \ , \ 2) = \text{‘’}]^{=}$

$[\text{vector-subseq} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 3 \ , \ 3) = \text{‘’}]^{=}$

$[\text{vector-subseq} ({}^{-}1000 \ , \ 0 \ , \ 3) = \text{‘’}]^{=}$

14.7 vector-length

$[\text{vector-length} (\text{‘’}) = 0]^{=}$

$[\text{vector-length} (\text{‘a’}) = 1]^{=}$

$[\text{vector-length} (\text{‘abc’}) = 3]^{=}$

$[\text{vector-length} (97 + 98 \cdot 256 + 2 \cdot 65536) = 2]^{=}$

$[\text{vector-length} ({}^{-}1000) = 0]^{=}$

14.8 vector-index

$[\text{vector-index} (\text{‘abc’} \ , \ {}^{-}1) = \bullet]^{=}$

$[\text{vector-index} (\text{‘abc’} \ , \ 0) = 97]^{=}$

$[\text{vector-index} (\text{‘abc’} \ , \ 1) = 98]^{=}$

$[\text{vector-index} (\text{‘abc’} \ , \ 2) = 99]^{=}$

$[\text{vector-index} (\text{‘abc’} \ , \ 3) = \bullet]^{=}$

$[\text{vector-index} (\text{‘’} \ , \ {}^{-}1) = \bullet]^{=}$

$[\text{vector-index} (\text{‘’} \ , \ 0) = \bullet]^{=}$

$[\text{vector-index} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ {}^{-}1) = \bullet]^{=}$

$[\text{vector-index} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 0) = 97]^{=}$

$[\text{vector-index} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 1) = 98]^{=}$

$[\text{vector-index} (97 + 98 \cdot 256 + 2 \cdot 65536 \ , \ 2) = \bullet]^{=}$

14.9 vector-head and vector-tail

$[\text{vector-head} (\text{`ABC'}) = 65]^{=}$

$[\text{vector-tail} (\text{`ABC'} ) = \text{`BC'}]^{=}$

14.10 vector2byte*

$[\text{vector2byte}{*} (\text{`'}) = \langle \rangle]^{=}$

$[\text{vector2byte}{*} (\text{`A'}) = \langle 65 \rangle]^{=}$

$[\text{vector2byte}{*} (\text{`ABC'}) = \langle 65, 66, 67 \rangle]^{=}$

14.11 vector2vector*

$[\text{vector2vector}{*} (\text{`'}) = \langle \rangle]^{=}$

$[\text{vector2vector}{*} (\text{`A'}) = \langle \mathrm{A} \rangle]^{=}$

$[\text{vector2vector}{*} (\text{`ABC'}) = \langle \mathrm{A}, \mathrm{B}, \mathrm{C} \rangle]^{=}$

14.12 bt2byte*

$[\text{bt2byte}{*} (65) = \langle 65 \rangle]^{=}$

$[\text{bt2byte}{*} ({}^{-}1) = \langle \rangle]^{=}$

$[\text{bt2byte}{*} (256) = \langle \rangle]^{=}$

$[\text{bt2byte}{*} (\mathsf{T}) = \langle \rangle]^{=}$

$[\text{bt2byte}{*} (\mathsf{F}) = \langle \rangle]^{=}$

$[\text{bt2byte}{*} (\text{map} (\lambda x.x)) = \langle \rangle]^{=}$

$[\text{bt2byte}{*} (65 :: 66) = \langle 65, 66 \rangle]^{=}$

$[\text{bt2byte}{*} ((65 :: 66) :: (67 :: 68)) = \langle 65, 66, 67, 68 \rangle]^{=}$

$[\text{bt2byte}{*} ((65 :: 66) :: \mathsf{T} :: 67 :: \mathsf{F} :: 68) = \langle 65, 66, 67, 68 \rangle]^{=}$

14.13 bt2vector*

$[\text{bt2vector}{*} (65) = \langle 65 + 256 \rangle]^{=}$

$[\text{bt2vector}{*} ({}^{-}1) = \langle \rangle]^{=}$

$[\text{bt2vector}{*} (256) = \langle \rangle]^{=}$

$[\text{bt2vector}{*} (\mathsf{T}) = \langle \rangle]^{=}$

$[\text{bt2vector}{*} (\mathsf{F}) = \langle \rangle]^{=}$

$[\text{bt2vector}* \ (\ \text{map} \ (\ \lambda x.x \ ) \ ) = \langle \, \rangle]^{=}$

$[\text{bt2vector}* \ (\ 65 :: 66 \ ) = \langle \text{A}, \text{B} \rangle]^{=}$

$[\text{bt2vector}* \ (\ (65 :: 66) :: (67 :: 68) \ ) = \langle \text{A}, \text{B}, \text{C}, \text{D} \rangle]^{=}$

$[\text{bt2vector}* \ (\ (65 :: 66) :: \mathsf{T} :: 67 :: \mathsf{F} :: 68 \ ) = \langle \text{A}, \text{B}, \text{C}, \text{D} \rangle]^{=}$

14.14 bt2vector

$[\text{bt2vector} \ (\ 65 \ ) = \text{`A'}]^{=}$

$[\text{bt2vector} \ (\ ^{-}1 \ ) = \text{`'}]^{=}$

$[\text{bt2vector} \ (\ 256 \ ) = \text{`'}]^{=}$

$[\text{bt2vector} \ (\ \mathsf{T} \ ) = \text{`'}]^{=}$

$[\text{bt2vector} \ (\ \mathsf{F} \ ) = \text{`'}]^{=}$

$[\text{bt2vector} \ (\ \text{map} \ (\ \lambda x.x \ ) \ ) = \text{`'}]^{=}$

$[\text{bt2vector} \ (\ 65 :: 66 \ ) = \text{`AB'}]^{=}$

$[\text{bt2vector} \ (\ (65 :: 66) :: (67 :: 68) \ ) = \text{`ABCD'}]^{=}$

$[\text{bt2vector} \ (\ (65 :: 66) :: \mathsf{T} :: 67 :: \mathsf{F} :: 68 \ ) = \text{`ABCD'}]^{=}$

14.15 vt2byte*

$[\text{vt2byte}* \ (\ \text{`'} \ ) = \langle \, \rangle]^{=}$

$[\text{vt2byte}* \ (\ \text{`A'} \ ) = \langle 65 \rangle]^{=}$

$[\text{vt2byte}* \ (\ \text{`AB'} \ ) = \langle 65, 66 \rangle]^{=}$

$[\text{vt2byte}* \ (\ ^{-}1 \ ) = \langle \, \rangle]^{=}$

$[\text{vt2byte}* \ (\ 65 + 2 \cdot 256 \ ) = \langle 65 \rangle]^{=}$

$[\text{vt2byte}* \ (\ \mathsf{T} \ ) = \langle \, \rangle]^{=}$

$[\text{vt2byte}* \ (\ \mathsf{F} \ ) = \langle \, \rangle]^{=}$

$[\text{vt2byte}* \ (\ \text{map} \ (\ \lambda x.x \ ) \ ) = \langle \, \rangle]^{=}$

$[\text{vt2byte}* \ (\ \text{`AB'} :: \text{CD} \ ) = \langle 65, 66, 67, 68 \rangle]^{=}$

$[\text{vt2byte}* \ (\ (\text{`A'} :: \text{`B'}) :: (\text{`C'} :: \text{D}) \ ) = \langle 65, 66, 67, 68 \rangle]^{=}$

$[\text{vt2byte}* \ (\ (\text{`AB'} :: \text{`C'}) :: \text{map} \ (\ \lambda x.x \ ) :: \mathsf{T} :: \text{`'} :: \text{D} \ ) = \langle 65, 66, 67, 68 \rangle]^{=}$

14.16 vt2vector*

$[\text{vt2vector}* \ (\ \text{`'}\ ) = \langle\,\rangle]^{=}$

$[\text{vt2vector}* \ (\ \text{`A'}\ ) = \langle \text{A} \rangle]^{=}$

$[\text{vt2vector}* \ (\ \text{`AB'}\ ) = \langle \text{A}, \text{B} \rangle]^{=}$

$[\text{vt2vector}* \ (\ {}^{-}1\ ) = \langle\,\rangle]^{=}$

$[\text{vt2vector}* \ (\ 65 + 2 \cdot 256\ ) = \langle \text{A} \rangle]^{=}$

$[\text{vt2vector}* \ (\ \mathsf{T}\ ) = \langle\,\rangle]^{=}$

$[\text{vt2vector}* \ (\ \mathsf{F}\ ) = \langle\,\rangle]^{=}$

$[\text{vt2vector}* \ (\ \text{map}\ (\ \lambda x.x\ )\ ) = \langle\,\rangle]^{=}$

$[\text{vt2vector}* \ (\ \text{`AB'} :: \text{CD}\ ) = \langle \text{A}, \text{B}, \text{C}, \text{D} \rangle]^{=}$

$[\text{vt2vector}* \ (\ (\text{`A'} :: \text{`B'}) :: (\text{`C'} :: \text{D})\ ) = \langle \text{A}, \text{B}, \text{C}, \text{D} \rangle]^{=}$

$[\text{vt2vector}* \ (\ (\text{`AB'} :: \text{`C'}) :: \text{map}\ (\ \lambda x.x\ ) :: \mathsf{T} :: \text{`'} :: \text{D}\ ) = \langle \text{A}, \text{B}, \text{C}, \text{D} \rangle]^{=}$

14.17 vt2vector

$[\text{vt2vector}\ (\ \text{`'}\ ) = \text{`'}]^{=}$

$[\text{vt2vector}\ (\ \text{`A'}\ ) = \text{`A'}]^{=}$

$[\text{vt2vector}\ (\ \text{`AB'}\ ) = \text{`AB'}]^{=}$

$[\text{vt2vector}\ (\ {}^{-}1\ ) = \text{`'}]^{=}$

$[\text{vt2vector}\ (\ 65 + 2 \cdot 256\ ) = \text{`A'}]^{=}$

$[\text{vt2vector}\ (\ \mathsf{T}\ ) = \text{`'}]^{=}$

$[\text{vt2vector}\ (\ \mathsf{F}\ ) = \text{`'}]^{=}$

$[\text{vt2vector}\ (\ \text{map}\ (\ \lambda x.x\ )\ ) = \text{`'}]^{=}$

$[\text{vt2vector}\ (\ \text{`AB'} :: \text{CD}\ ) = \text{`ABCD'}]^{=}$

$[\text{vt2vector}\ (\ (\text{`A'} :: \text{`B'}) :: (\text{`C'} :: \text{D})\ ) = \text{`ABCD'}]^{=}$

$[\text{vt2vector}\ (\ (\text{`AB'} :: \text{`C'}) :: \text{map}\ (\ \lambda x.x\ ) :: \mathsf{T} :: \text{`'} :: \text{D}\ ) = \text{`ABCD'}]^{=}$

15 Structures

15.1 List access

$[2 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{0}]^{\cdot}$

$[3 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{1}]^{\cdot}$

$[4 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{2}]^{\cdot}$

$[5 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{3}]^{\cdot}$

$[6 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{4}]^{\cdot}$

$[7 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{5}]^{\cdot}$

$[8 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{6}]^{\cdot}$

$[9 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{7}]^{\cdot}$

$[0 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{8}]^{\cdot}$

$[1 = (2::3::4::5::6::7::8::9::0::1::\mathsf{T})^{9}]^{\cdot}$

15.2 Tree access

$[\lceil \mathrm{base} \rceil^{\mathrm{r}} = \lceil x + y \rceil^{\mathrm{r}}]^{\cdot}$

$[\lceil \mathrm{base} \rceil^{\mathrm{i}} = 0]^{\cdot}$

$[\lceil \mathrm{abc} \rceil^{\mathrm{r}} = 0]^{\cdot}$

$[\lceil \mathrm{abc} \rceil^{\mathrm{i}} = \mathrm{abc}]^{\cdot}$

15.3 Tree equality

$[\lceil x + y \rceil \stackrel{\mathrm{t}}{=} \lceil x + y \rceil]^{\cdot}$

$[\lceil x + y \rceil \stackrel{\mathrm{t}}{=} \lceil z + y \rceil]^{-}$

$[\lceil x + y \rceil \stackrel{\mathrm{t}}{=} \lceil x + z \rceil]^{-}$

$[\lceil x + y \rceil \stackrel{\mathrm{t}}{=} \lceil x - y \rceil]^{-}$

$[1 = \mathrm{lookup}\ (\ \lceil x \rceil\ ,\ (\lceil x \rceil :: 1) :: (\lceil y_1 \rceil :: 2) :: (\lceil y_2 \rceil :: 3) :: \mathsf{T}\ ,\ \mathsf{T}\ )]^{\cdot}$

$[2 = \mathrm{lookup}\ (\ \lceil y_1 \rceil\ ,\ (\lceil x \rceil :: 1) :: (\lceil y_1 \rceil :: 2) :: (\lceil y_2 \rceil :: 3) :: \mathsf{T}\ ,\ \mathsf{T}\ )]^{\cdot}$

$[3 = \mathrm{lookup}\ (\ \lceil y_2 \rceil\ ,\ (\lceil x \rceil :: 1) :: (\lceil y_1 \rceil :: 2) :: (\lceil y_2 \rceil :: 3) :: \mathsf{T}\ ,\ \mathsf{T}\ )]^{\cdot}$

$[\mathsf{T} = \mathrm{lookup}\ (\ \lceil y_3 \rceil\ ,\ (\lceil x \rceil :: 1) :: (\lceil y_1 \rceil :: 2) :: (\lceil y_2 \rceil :: 3) :: \mathsf{T}\ ,\ \mathsf{T}\ )]^{\cdot}$

$[\mathsf{F} = \mathrm{lookup}\ (\ \lceil y_3 \rceil\ ,\ (\lceil x \rceil :: 1) :: (\lceil y_1 \rceil :: 2) :: (\lceil y_2 \rceil :: 3) :: \mathsf{T}\ ,\ \mathsf{F}\ )]^{\cdot}$

15.4 Arrays

$[\mathsf{T}[2]]^{\cdot}$

$[12 = \mathsf{T}[2{\to}12][2]]^{\cdot}$

$[\mathsf{T}[2{\to}12][3]]^{\cdot}$

$[12 = \mathsf{T}[2{\to}12][3{\to}13][2]]^{\cdot}$

$[13 = \mathsf{T}[2{\to}12][3{\to}13][3]]^{\cdot}$

$[\mathsf{T}[2{\to}12][3{\to}13][4]]^{\cdot}$

$[12 = \mathsf{T}[2{\to}12][3{\to}13][10{\to}14][2]]^{\cdot}$

$[13 = \mathsf{T}[2{\to}12][3{\to}13][10{\to}14][3]]^{\cdot}$

$[14 = \mathsf{T}[2{\to}12][3{\to}13][10{\to}14][10]]^{\cdot}$

$[\mathsf{T}[2{\to}12][3{\to}13][10{\to}14][5]]^{\cdot}$

$[\mathsf{T}[2{\to}12][3{\to}13][10{\to}14][2{\to}\mathsf{T}] = \mathsf{T}[10{\to}14][3{\to}13]]^{\cdot}$

$[\mathsf{T}[2::3::4::\mathsf{T}{\Rightarrow}5][2::4::\mathsf{T}{\Rightarrow}6][2][3][4] = 5]^{=}$

$[\mathsf{T}[2::3::4::\mathsf{T}{\Rightarrow}5][2::4::\mathsf{T}{\Rightarrow}6][2][4] = 6]^{=}$

$[\mathsf{T}[2::3::4::\mathsf{T}{\Rightarrow}5][2::4::\mathsf{T}{\Rightarrow}6][2][5] = \mathsf{T}]^{=}$

$[\mathsf{T}[\mathrm{code}{\to}3][\mathrm{codex}{\to}4][\mathrm{code}] = 3]^{=}$

$[\mathsf{T}[\mathrm{code}::\mathsf{T}{\Rightarrow}3][\mathrm{codex}::\mathsf{T}{\Rightarrow}4][\mathrm{code}] = 3]^{=}$

$[\mathsf{T}[1::\mathrm{code}::\mathsf{T}{\Rightarrow}3][1::\mathrm{codex}::\mathsf{T}{\Rightarrow}4][1][\mathrm{code}] = 3]^{=}$

$[\mathsf{T}[1::\mathrm{code}::2::\mathsf{T}{\Rightarrow}3][1::\mathrm{codex}::2::\mathsf{T}{\Rightarrow}4][1][\mathrm{code}][2] = 3]^{=}$

16 Evaluation

16.1 The page symbol

$[\mathrm{base}[0] = \lceil \mathrm{base} \rceil^{\mathrm{r}}]^{\cdot}$

$[\mathrm{base}[\lceil \mathrm{base} \rceil^{\mathrm{r}}][\mathrm{vector}] \in \mathbf{Z}]^{\cdot}$

$[\mathrm{base}[\lceil \mathrm{base} \rceil^{\mathrm{r}}][\mathrm{bibliography}] = \lceil \mathrm{base} \rceil^{\mathrm{r}} :: \mathsf{T}]^{\cdot}$

$[\mathrm{base}[\lceil \mathrm{base} \rceil^{\mathrm{r}}][\mathrm{dictionary}][\lceil x + y \rceil^{\mathrm{i}}] = 2]^{\cdot}$

$[\mathrm{base}[\lceil \mathrm{base} \rceil^{\mathrm{r}}][\mathrm{body}] \in \mathbf{P}]^{\cdot}$

$[\mathrm{base}[\lceil \mathrm{base} \rceil^{\mathrm{r}}][\mathrm{codex}][\lceil \mathsf{F} \rceil^{\mathrm{r}}][\lceil \mathsf{F} \rceil^{\mathrm{i}}][0][\mathrm{value}] \stackrel{\mathrm{t}}{=} \lceil [\mathsf{F} \xrightarrow{\mathrm{val}} \mathsf{T}\ \mathrm{LazyPair}\ \mathsf{T}] \rceil]^{\cdot}$

$[\text{base}[\lceil \text{base}\rceil^{\text{r}}][\text{expansion}] \in \mathbf{P}]^{\cdot}$

$[\text{base}[\lceil \text{base}\rceil^{\text{r}}][\text{code}][\lceil x - y\rceil^{\text{i}}] \text{ Tail} \,' \,2 \,' \,3 = {}^{-}1]^{\cdot}$

$[\text{base}[\lceil \text{base}\rceil^{\text{r}}][\text{cluster}] = \mathsf{T}]^{\cdot}$

$[\text{base}[\lceil \text{base}\rceil^{\text{r}}][\text{diagnose}]^{\text{U}} = \mathsf{T}]^{\cdot}$

16.2 eval

$[2 = \text{eval} \ (\ \lceil 2\rceil \ , \ \mathsf{T} \ , \ \text{base} \ )^{\text{U}}]^{\cdot}$

$[{}^{-}1 = \text{eval} \ (\ \lceil 2 - 3\rceil \ , \ \mathsf{T} \ , \ \text{base} \ )^{\text{U}}]^{\cdot}$

$[\text{abc} = \text{eval} \ (\ \lceil \text{abc}\rceil \ , \ \mathsf{T} \ , \ \text{base} \ )^{\text{U}}]^{\cdot}$

$[\lceil x + y\rceil^{\text{r}} = \text{eval} \ (\ \lceil \text{base}[0]\rceil \ , \ \mathsf{T} \ , \ \text{base} \ )^{\text{U}}]^{\cdot}$

$[\lceil 2 - 3\rceil \stackrel{\text{t}}{=} \text{eval} \ (\ \lceil\lceil 2 - 3\rceil\rceil \ , \ \mathsf{T} \ , \ \text{base} \ )^{\text{U}}]^{\cdot}$

$[{}^{-}1 = \text{eval} \ (\ \lceil \lambda x.x - 3\rceil \ , \ \mathsf{T} \ , \ \text{base} \ ) \text{ Tail} \,' \,2]^{\cdot}$

$[{}^{-}1 = \text{eval} \ (\ \lceil (\lambda x.x - 3) \,' \,2\rceil \ , \ \mathsf{T} \ , \ \text{base} \ )^{\text{U}}]^{\cdot}$

$[6 = \text{eval} \ (\ \lceil (\lambda x.x \,' \,x) \,' \,(\lambda f.\lambda x.f \,' \,(f \,' \,x)) \,' \,(\lambda x.x + 1) \,' \,2\rceil \ , \ \mathsf{T} \ , \ \text{base} \ )^{\text{U}}]^{\cdot}$

17 Macro expansion

17.1 Parentheses

$[\lceil (2)\rceil \stackrel{\text{t}}{=} \lceil 2\rceil]^{\cdot}$

$[\lceil$

$2\rceil \stackrel{\text{t}}{=} \lceil 2\rceil]^{\cdot}$

17.2 Tuples

$[\lceil \langle 2, 3, 4\rangle\rceil \stackrel{\text{t}}{=} \lceil 2 :: 3 :: 4 :: \langle \, \rangle\rceil]^{\cdot}$

17.3 Eager definitions

$[\lceil [x + y \stackrel{\bullet}{=} x \cdot y]\rceil \stackrel{\text{t}}{=} \lceil [x + y \stackrel{\text{val}}{\to} \text{norm } x {\in} \mathbf{V} \colon y {\in} \mathbf{V} \colon x \cdot y]\rceil]^{\cdot}$

17.4 Let

$[\lceil \mathbf{let}\ 3 \ddot{=} 4\ \mathbf{in}\ 2\rceil \stackrel{t}{=} \lceil 2\rceil]^{\cdot}$

$[\lceil \mathbf{let}\ 3 \ddot{=} 4\ \mathbf{in}\ 3\rceil \stackrel{t}{=} \lceil 4\rceil]^{\cdot}$

$[\lceil \mathbf{let}\ 3 \ddot{=} 4\ \mathbf{in}\ 4\rceil \stackrel{t}{=} \lceil 4\rceil]^{\cdot}$

$[\text{make-var}\ (\ \lceil x\rceil\ ) \stackrel{t}{=} \lceil *\rceil]^{\cdot}$

$[\text{make-let}\ (\ \lceil x\rceil\ ,\ \lceil y\rceil\ ,\ \lceil z\rceil\ ) \stackrel{t}{=} \lceil \mathbf{let}\ y\ \mathbf{be}\ x\ \mathbf{in}\ z\rceil]^{\cdot}$

$[\text{make-prime}\ (\ \lceil x\rceil\ ) \stackrel{t}{=} \lceil x'\rceil]^{\cdot}$

$[\text{make-head}\ (\ \lceil x\rceil\ ) \stackrel{t}{=} \lceil x^{\text{h}}\rceil]^{\cdot}$

$[\text{make-tail}\ (\ \lceil x\rceil\ ) \stackrel{t}{=} \lceil x^{\text{t}}\rceil]^{\cdot}$

$[\text{let1}\ (\ \text{protect}(\lceil \mathbf{let}\ x = y\ \mathbf{in}\ z\rceil) :: \text{macrostate0} :: \text{self} :: \mathsf{T}\ ) \stackrel{t}{=} \lceil \mathbf{let}\ y\ \mathbf{be}\ *\ \mathbf{in}\ \mathbf{let}\ *\ \mathbf{be}\ x\ \mathbf{in}\ z\rceil]^{\cdot}$

$[\text{let1}\ (\ \text{protect}(\lceil \mathbf{let}\ 2 = y\ \mathbf{in}\ z\rceil) :: \text{macrostate0} :: \text{self} :: \mathsf{T}\ ) \stackrel{t}{=} \lceil \mathbf{let}\ y\ \mathbf{be}\ *\ \mathbf{in}\ z\rceil]^{\cdot}$

$[\text{let1}\ (\ \text{protect}(\lceil \mathbf{let}\ u :: v = y\ \mathbf{in}\ z\rceil) :: \text{macrostate0} :: \text{self} :: \mathsf{T}\ ) \stackrel{t}{=} \lceil$
$\mathbf{let}\ y\ \mathbf{be}\ *\ \mathbf{in}$
$\mathbf{let}\ (\mathbf{if}\ * \in \mathbf{A}\ \mathbf{then}\ *\ \mathbf{else}\ *^{\text{h}}) :: (\mathbf{if}\ * \in \mathbf{A}\ \mathbf{then}\ *\ \mathbf{else}\ *^{\text{t}}) :: \mathsf{T}\ \mathbf{be}\ *'\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{h}}\ \mathbf{be}\ *\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{t}}\ \mathbf{be}\ *'\ \mathbf{in}$
$\mathbf{let}\ *\ \mathbf{be}\ u\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{h}}\ \mathbf{be}\ *\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{t}}\ \mathbf{be}\ *'\ \mathbf{in}$
$\mathbf{let}\ *\ \mathbf{be}\ v\ \mathbf{in}\ z\rceil]^{\cdot}$

$[\lceil \mathbf{let}\ x = y\ \mathbf{in}\ z\rceil \stackrel{t}{=} \lceil \mathbf{let}\ y\ \mathbf{be}\ *\ \mathbf{in}\ \mathbf{let}\ *\ \mathbf{be}\ x\ \mathbf{in}\ z\rceil]^{\cdot}$

$[\lceil \mathbf{let}\ 2 = y\ \mathbf{in}\ z\rceil \stackrel{t}{=} \lceil \mathbf{let}\ y\ \mathbf{be}\ *\ \mathbf{in}\ z\rceil]^{\cdot}$

$[\lceil \mathbf{let}\ u :: v = y\ \mathbf{in}\ z\rceil \stackrel{t}{=} \lceil$
$\mathbf{let}\ y\ \mathbf{be}\ *\ \mathbf{in}$
$\mathbf{let}\ (\mathbf{if}\ * \in \mathbf{A}\ \mathbf{then}\ *\ \mathbf{else}\ *^{\text{h}}) :: (\mathbf{if}\ * \in \mathbf{A}\ \mathbf{then}\ *\ \mathbf{else}\ *^{\text{t}}) :: \mathsf{T}\ \mathbf{be}\ *'\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{h}}\ \mathbf{be}\ *\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{t}}\ \mathbf{be}\ *'\ \mathbf{in}$
$\mathbf{let}\ *\ \mathbf{be}\ u\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{h}}\ \mathbf{be}\ *\ \mathbf{in}$
$\mathbf{let}\ *'^{\text{t}}\ \mathbf{be}\ *'\ \mathbf{in}$
$\mathbf{let}\ *\ \mathbf{be}\ v\ \mathbf{in}\ z\rceil]^{\cdot}$

$[\mathbf{let}\ u = 2\ \mathbf{in}\ u = 2]^{=}$

$[\mathbf{let}\ u :: v = 2\ \mathbf{in}\ u = 2]^{=}$

$[\mathbf{let}\ u :: v = 2\ \mathbf{in}\ v = 2]^{=}$

$[\mathbf{let}\ u :: v = 3\ \mathbf{in}\ v = 3]^{=}$

$[\mathbf{let}\ u :: v = 2 :: 3\ \mathbf{in}\ u = 2]^{=}$

$[\mathbf{let}\ u :: v = 2 :: 3\ \mathbf{in}\ v = 3]^{=}$

$[\mathbf{let}\ u :: v = 2 :: 3\ \mathbf{in}\ 4 = 4]^{=}$

$[\mathbf{let}\ (u) = 2\ \mathbf{in}\ u = 2]^{=}$

$[\mathbf{let}\ ((u :: v) :: (w :: x)) = ((1 :: 2) :: (3 :: 4))\ \mathbf{in}\ u = 1]^{=}$

$[\mathbf{let}\ ((u :: v) :: (w :: x)) = ((1 :: 2) :: (3 :: 4))\ \mathbf{in}\ v = 2]^{=}$

$[\mathbf{let}\ ((u :: v) :: (w :: x)) = ((1 :: 2) :: (3 :: 4))\ \mathbf{in}\ w = 3]^{=}$

$[\mathbf{let}\ ((u :: v) :: (w :: x)) = ((1 :: 2) :: (3 :: 4))\ \mathbf{in}\ x = 4]^{=}$

$[\mathbf{let}\ \langle u, v, w\rangle = \langle 1, 2, 3\rangle\ \mathbf{in}\ \langle v, w, u\rangle = \langle 2, 3, 1\rangle]^{=}$

17.5 Backquote

$[\text{make-pair}\ (\ 1\ ,\ \lceil 2\rceil\ ,\ \lceil 3\rceil\ ) \overset{\text{t}}{=} \lceil 2 :: 3\rceil]^{\cdot}$

$[\text{make-quote}\ (\ 1\ ,\ \lceil 2\rceil\ ) \overset{\text{t}}{=} \lceil\lceil 2\rceil\rceil]^{\cdot}$

$[\text{make-make-root}\ (\ 1\ ,\ \lceil 2\rceil\ ,\ \lceil 3\rceil\ ) \overset{\text{t}}{=} \lceil\text{make-root}\ (\ 2\ ,\ 3\ )\rceil]^{\cdot}$

$[\text{backquote2}\ (\ (\mathsf{T} :: \mathsf{T} :: 1) :: \mathsf{T}\ ,\ (2 :: 3 :: \mathsf{T}) :: \mathsf{T}\ ,\ (4 :: 5 :: \mathsf{T}) :: \mathsf{T}\ ,\ \text{macrostate0}\ ,$
$\text{self}\ ) = ((\lceil \mathsf{T} :: \mathsf{T}\rceil^{\text{r}} :: \lceil \mathsf{T} :: \mathsf{T}\rceil^{\text{i}} :: 1) :: ((\lceil\text{make-root}\ (\ \mathsf{T}\ ,\ \mathsf{T}\ )\rceil^{\text{r}} :: \lceil\text{make-roo}$
$(\ \mathsf{T}\ ,\ \mathsf{T}\ )\rceil^{\text{i}} :: 1) :: ((2 :: 3 :: \mathsf{T}) :: \mathsf{T}) :: (((\lceil\lceil \mathsf{T}\rceil\rceil^{\text{r}} :: \lceil\lceil \mathsf{T}\rceil\rceil^{\text{i}} :: 1) :: ((4 :: 5 :: \mathsf{T}) ::$

$[\text{backquote2}\ (\ \lceil 1\rceil\ ,\ \lceil 2\rceil\ ,\ \lceil 3\rceil\ ,\ \text{macrostate0}\ ,\ \text{self}\ ) \overset{\text{t}}{=} \lceil\text{make-root}\ (\ 2\ ,\ \lceil 3\rceil$
$) :: \mathsf{T}\rceil]^{\cdot}$

$[\text{backquote2}\ (\ \lceil 1\rceil\ ,\ \lceil 2\rceil\ ,\ \lceil \underline{3}\rceil\ ,\ \text{macrostate0}\ ,\ \text{self}\ ) \overset{\text{t}}{=} \lceil 3\rceil]^{\cdot}$

$[\lceil\lceil_{2} 3\rceil\rceil \overset{\text{t}}{=} \lceil\text{make-root}\ (\ 2\ ,\ \lceil 3\rceil\ ) :: \mathsf{T}\rceil]^{\cdot}$

$[\lceil\lceil_{2} \underline{\lceil 3\rceil}\rceil\rceil \overset{\text{t}}{=} \lceil\lceil 3\rceil\rceil]^{\cdot}$

$[\lceil_{\lceil 2\rceil} 3\rceil \overset{\text{t}}{=} \lceil 3\rceil]^{\cdot}$

$[\lceil_{\lceil 2\rceil} \underline{\lceil 3\rceil}\rceil \overset{\text{t}}{=} \lceil 3\rceil]^{\cdot}$

$[\mathbf{let}\ x = \lceil 3\rceil\ \mathbf{in}\ \lceil_{\lceil 2\rceil}\, \underline{x}\rceil \stackrel{\mathrm{t}}{=} \lceil 3\rceil]^{\cdot}$

$[\lceil_{\lceil 2\rceil}\, 3+4\rceil \stackrel{\mathrm{t}}{=} \lceil 3+4\rceil]^{\cdot}$

$[\lceil_{\lceil 2\rceil}\, 3+4\rceil \stackrel{\mathrm{t}}{=} \lceil 3+4\rceil]^{\cdot}$

$[\mathbf{let}\ x\ \mathbf{be}\ \lceil 1\rceil\ \mathbf{in}\ \mathbf{let}\ y\ \mathbf{be}\ \lceil 2\rceil\ \mathbf{in}\ \lceil_{x}\, 3+4\rceil \stackrel{\mathrm{t}}{=} \lceil 3+4\rceil]^{\cdot}$

$[\mathbf{let}\ x = \lceil 1\rceil\ \mathbf{in}\ \mathbf{let}\ y = \lceil 2\rceil\ \mathbf{in}\ \lceil_{x}\, \underline{y}+4\rceil \stackrel{\mathrm{t}}{=} \lceil 2+4\rceil]^{\cdot}$

$[\mathbf{let}\ x = \lceil 1\rceil\ \mathbf{in}\ \mathbf{let}\ y = \lceil 2\rceil\ \mathbf{in}\ \lceil_{x}\, 3+\underline{y}\rceil \stackrel{\mathrm{t}}{=} \lceil 3+2\rceil]^{\cdot}$

$[\mathbf{let}\ x = \lceil 1\rceil\ \mathbf{in}\ \mathbf{let}\ y = \lceil 2\rceil\ \mathbf{in}\ \lceil_{x}\, 3+\underline{y}\rceil^{\mathrm{d}} = x^{\mathrm{d}}]^{\cdot}$

18 Proof checking

18.1 Term sets

$[\lceil 2\rceil \in \langle\lceil 2\rceil, \lceil 3\rceil\rangle]^{\cdot}$

$[\lceil 3\rceil \in \langle\lceil 2\rceil, \lceil 3\rceil\rangle]^{\cdot}$

$[\lceil 4\rceil \in \langle\lceil 2\rceil, \lceil 3\rceil\rangle]^{-}$

$[\langle\lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set+}\ \lceil 2\rceil \stackrel{\mathrm{t}^*}{=} \langle\lceil 2\rceil, \lceil 3\rceil\rangle]^{\cdot}$

$[\langle\lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set+}\ \lceil 3\rceil \stackrel{\mathrm{t}^*}{=} \langle\lceil 2\rceil, \lceil 3\rceil\rangle]^{\cdot}$

$[\langle\lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set+}\ \lceil 4\rceil \stackrel{\mathrm{t}^*}{=} \langle\lceil 4\rceil, \lceil 2\rceil, \lceil 3\rceil\rangle]^{\cdot}$

$[\langle\lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set\text{-}}\ \lceil 2\rceil \stackrel{\mathrm{t}^*}{=} \langle\lceil 3\rceil\rangle]^{\cdot}$

$[\langle\lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set\text{-}}\ \lceil 3\rceil \stackrel{\mathrm{t}^*}{=} \langle\lceil 2\rceil\rangle]^{\cdot}$

$[\langle\lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set\text{-}}\ \lceil 4\rceil \stackrel{\mathrm{t}^*}{=} \langle\lceil 2\rceil, \lceil 3\rceil\rangle]^{\cdot}$

$[\langle 1, 2, 3\rangle\ \mathrm{union}\ \langle 4, 2\rangle = \langle 3, 1, 4, 2\rangle]^{\cdot}$

$[\langle\lceil 1\rceil, \lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set=}\ \langle\lceil 3\rceil, \lceil 1\rceil, \lceil 2\rceil\rangle]^{\cdot}$

$[\langle\lceil 1\rceil, \lceil 2\rceil, \lceil 3\rceil\rangle\ \mathrm{set=}\ \langle\lceil 3\rceil, \lceil 1\rceil\rangle]^{-}$

18.2 Meta variables

$[\lceil \mathsf{a}\rceil\ \mathrm{metavar}\ (\ \mathrm{self}\ )]^{\cdot}$

$[\lceil x\rceil\ \mathrm{metavar}\ (\ \mathrm{self}\ )]^{-}$

18.3 Object terms

$[\lceil \mathsf{a} \rceil$ objectterm (self $)]^{\cdot}$

$[\lceil \mathsf{a} \vdash \mathsf{b} \rceil$ objectterm (self $)]^{-}$

$[\lceil x \rceil$ objectterm (self $)]^{\cdot}$

$[\lceil x_{y \vdash z} \rceil$ objectterm (self $)]^{\cdot}$

$[\lceil \lceil x \vdash y \rceil \rceil$ objectterm (self $)]^{\cdot}$

$[\lceil \lambda x.y \rceil$ objectterm (self $)]^{\cdot}$

$[\lceil \lambda x.(y \vdash z) \rceil$ objectterm (self $)]^{-}$

$[\lceil \lambda (x \vdash y).z \rceil$ objectterm (self $)]^{-}$

$[\lceil x + y \rceil$ objectterm (self $)]^{\cdot}$

$[\lceil (x \vdash y) + z \rceil$ objectterm (self $)]^{-}$

$[\lceil 1 + (x \vdash y) \rceil$ objectterm (self $)]^{-}$

18.4 Meta terms

$[\lceil \mathsf{a} \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil \mathsf{a} \vdash \mathsf{b} \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil x \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil x_{y \vdash z} \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil \lceil x \vdash y \rceil \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil \lambda x.y \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil \lambda x.(y \vdash z) \rceil$ metaterm (self $)]^{-}$

$[\lceil \lambda (x \vdash y).z \rceil$ metaterm (self $)]^{-}$

$[\lceil \lambda 2.z \rceil$ metaterm (self $)]^{-}$

$[\lceil x + y \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil (x \vdash y) + z \rceil$ metaterm (self $)]^{-}$

$[\lceil 1 + (x \vdash y) \rceil$ metaterm (self $)]^{-}$

$[\lceil (\mathsf{a} \vdash x) \vdash (1 \vdash 2) \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil (1 + 2 + 3) \vdash (4 + 5 + 6) \rceil$ metaterm (self $)]^{\cdot}$

$[\lceil ((1 \vdash 2) + 3) \vdash (4 + 5 + 6) \rceil$ metaterm (self $)]^{-}$

$[\lceil(1 + (2 \vdash 3)) \vdash (4 + 5 + 6)\rceil \text{ metaterm (self)}]^{-}$

$[\lceil(1 + 2 + 3) \vdash ((4 \vdash 5) + 6)\rceil \text{ metaterm (self)}]^{-}$

$[\lceil(1 + 2 + 3) \vdash (4 + (5 \vdash 6))\rceil \text{ metaterm (self)}]^{-}$

$[\lceil\Pi \mathsf{a}\colon \mathsf{a} \vdash \mathsf{a}\rceil \text{ metaterm (self)}]^{\cdot}$

$[\lceil\Pi(1 \vdash 2) + 3\colon 4\rceil \text{ metaterm (self)}]^{-}$

$[\lceil\Pi 1\colon (2 \vdash 3) + 4\rceil \text{ metaterm (self)}]^{-}$

18.5 Avoidance

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{b}\rceil]^{\cdot}$

$[\lceil x\rceil \text{ metaavoid (self) } \lceil b\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{a} \vdash \mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{b} \vdash \mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{a} \vdash \mathsf{c}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{b} \vdash \mathsf{c}\rceil]^{\cdot}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\Pi \mathsf{a}\colon \mathsf{a}\rceil]^{\cdot}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\Pi \mathsf{a}\colon \mathsf{b}\rceil]^{\cdot}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\Pi \mathsf{b}\colon \mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\Pi \mathsf{b}\colon \mathsf{b}\rceil]^{\cdot}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{a} + \mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{b} + \mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{a} + \mathsf{c}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\mathsf{b} + \mathsf{c}\rceil]^{\cdot}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\lambda \mathsf{a}.\mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\lambda \mathsf{b}.\mathsf{a}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\lambda \mathsf{a}.\mathsf{c}\rceil]^{-}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\lambda \mathsf{b}.\mathsf{c}\rceil]^{\cdot}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil\lceil\mathsf{a}\rceil\rceil]^{\cdot}$

$[\lceil\mathsf{a}\rceil \text{ metaavoid (self) } \lceil(\Pi \mathsf{a}\colon \mathsf{a}) \vdash (\mathsf{b} \vdash \lceil\mathsf{a}\rceil) \Vdash \perp\!\!\!\perp\rceil]^{\cdot}$

$[\lceil \mathsf{a} \rceil \text{ metaavoid (self) } \lceil (\Pi \mathsf{a} \colon \mathsf{a}) \vdash (\mathsf{a} \vdash \lceil \mathsf{a} \rceil) \Vdash \perp\!\!\!\perp \rceil]^{-}$

$[\lceil \mathsf{a} \rceil \text{ metaavoid* (self) } \langle \lceil \mathsf{b}+\mathsf{c} \rceil, \lceil \mathsf{d}+\mathsf{e} \rceil \rangle]^{\cdot}$

$[\lceil \mathsf{b} \rceil \text{ metaavoid* (self) } \langle \lceil \mathsf{b}+\mathsf{c} \rceil, \lceil \mathsf{d}+\mathsf{e} \rceil \rangle \stackrel{\mathrm{t}}{=} \lceil \mathsf{b}+\mathsf{c} \rceil]^{\cdot}$

$[\lceil \mathsf{c} \rceil \text{ metaavoid* (self) } \langle \lceil \mathsf{b}+\mathsf{c} \rceil, \lceil \mathsf{d}+\mathsf{e} \rceil \rangle \stackrel{\mathrm{t}}{=} \lceil \mathsf{b}+\mathsf{c} \rceil]^{\cdot}$

$[\lceil \mathsf{d} \rceil \text{ metaavoid* (self) } \langle \lceil \mathsf{b}+\mathsf{c} \rceil, \lceil \mathsf{d}+\mathsf{e} \rceil \rangle \stackrel{\mathrm{t}}{=} \lceil \mathsf{d}+\mathsf{e} \rceil]^{\cdot}$

$[\lceil \mathsf{e} \rceil \text{ metaavoid* (self) } \langle \lceil \mathsf{b}+\mathsf{c} \rceil, \lceil \mathsf{d}+\mathsf{e} \rceil \rangle \stackrel{\mathrm{t}}{=} \lceil \mathsf{d}+\mathsf{e} \rceil]^{\cdot}$

$[\text{distinctvar (} \langle \rangle \text{ , self)}]^{\cdot}$

$[\text{distinctvar (} \langle \lceil x \rceil \rangle \text{ , self)}]^{\cdot}$

$[\text{distinctvar (} \langle \lceil x \rceil, \lceil y \rceil \rangle \text{ , self)}]^{\cdot}$

$[\text{distinctvar (} \langle \lceil x \rceil, \lceil y \rceil, \lceil z \rceil \rangle \text{ , self)}]^{\cdot}$

$[\text{distinctvar (} \langle \lceil x \rceil, \lceil \mathsf{y} \rceil, \lceil z \rceil \rangle \text{ , self)}]^{-}$

$[\text{distinctvar (} \langle \lceil x \rceil, \lceil y \rceil, \lceil x \rceil \rangle \text{ , self)}]^{-}$

$[\text{distinctvar (} \langle \lceil x \rceil, \lceil y \rceil, \lceil y \rceil \rangle \text{ , self)}]^{-}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil x \rceil]^{-}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil y \rceil]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil x+x \rceil]^{-}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil y+x \rceil]^{-}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil x+z \rceil]^{-}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil y+z \rceil]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \lambda x.x \rceil]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \lambda y.x \rceil]^{-}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \lambda x.y \rceil]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \lambda y.y \rceil]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \lceil x \rceil \rceil]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \mathsf{a} \rceil \text{ set= } \langle \lceil \mathsf{a} \rceil \rangle]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \mathsf{a}+\mathsf{b} \rceil \text{ set= } \langle \lceil \mathsf{a} \rceil, \lceil \mathsf{b} \rceil \rangle]^{\cdot}$

$[\lceil x \rceil \text{ objectavoid (self) } \lceil \lambda x.\mathsf{a} \rceil \text{ set= } \langle \rangle]^{\cdot}$

$[\lceil x\rceil$ objectavoid (self) $\lceil\lambda y.\mathsf{a}\rceil$ set= $\langle\lceil\mathsf{a}\rceil\rangle]^{\cdot}$

$[\lceil x\rceil$ objectavoid (self) $\lceil\lceil\mathsf{a}\rceil\rceil$ set= $\langle\,\rangle]^{\cdot}$

$[\lceil x\rceil$ objectavoid (self) $\lceil\lambda\mathsf{a}.x\rceil]^{-}$.

$[\langle\lceil x\rceil,\lceil y\rceil\rangle$ objectavoid$*$ (self) $\lceil x\rceil]^{-}$.

$[\langle\lceil x\rceil,\lceil y\rceil\rangle$ objectavoid$*$ (self) $\lceil y\rceil]^{-}$.

$[\langle\lceil x\rceil,\lceil y\rceil\rangle$ objectavoid$*$ (self) $\lceil z\rceil]^{\cdot}$.

$[\langle\lceil x\rceil,\lceil y\rceil\rangle$ objectavoid$*$ (self) $\lceil x+y\rceil]^{-}$.

$[\langle\lceil x\rceil,\lceil y\rceil\rangle$ objectavoid$*$ (self) $\lceil\lambda x.x+y\rceil]^{-}$.

$[\langle\lceil x\rceil,\lceil y\rceil\rangle$ objectavoid$*$ (self) $\lceil\lambda y.x+y\rceil]^{-}$.

$[\langle\lceil x\rceil,\lceil y\rceil\rangle$ objectavoid$*$ (self) $\lceil\lambda x.\lambda y.x+y\rceil]^{\cdot}$.

$[\langle\lceil x\rceil,\lceil v\rceil\rangle$ objectavoid$*$ (self) $\lceil y\rceil]^{\cdot}$.

18.6 Sequent equality

$[\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle$ sequent= $\langle\langle\lceil 2\rceil,\lceil 1\rceil\rangle,\langle\lceil 4\rceil,\lceil 3\rceil\rangle,\lceil 5\rceil\rangle]^{\cdot}$

$[\langle\langle\lceil 6\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle$ sequent= $\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle]^{-}$

$[\langle\langle\lceil 1\rceil,\lceil 6\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle$ sequent= $\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle]^{-}$

$[\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 6\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle$ sequent= $\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle]^{-}$

$[\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 6\rceil\rangle,\lceil 5\rceil\rangle$ sequent= $\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle]^{-}$

$[\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 6\rceil\rangle$ sequent= $\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle]^{-}$

18.7 Substitution freeness

[metafree ($\lceil\mathsf{a}\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\mathsf{a}\rceil$, self)$]^{\cdot}$

[metafree ($\lceil\mathsf{a}\vdash\mathsf{b}\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\Pi\mathsf{c}\colon(\mathsf{c}\vdash\mathsf{b})\rceil$, self)$]^{\cdot}$

[metafree ($\lceil\Pi\mathsf{a}\colon(\mathsf{a}\vdash\mathsf{b})\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\Pi\mathsf{c}\colon(\mathsf{c}\vdash\mathsf{b})\rceil$, self)$]^{\cdot}$

[metafree ($\lceil\Pi\mathsf{b}\colon\mathsf{a}\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\Pi\mathsf{c}\colon(\mathsf{c}\vdash\mathsf{b})\rceil$, self)$]^{-}$

[metafree ($\lceil\Pi\mathsf{b}\colon\mathsf{b}\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\Pi\mathsf{c}\colon(\mathsf{c}\vdash\mathsf{b})\rceil$, self)$]^{\cdot}$

[metafree ($\lceil\Pi\mathsf{c}\colon\mathsf{a}\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\Pi\mathsf{c}\colon(\mathsf{c}\vdash\mathsf{b})\rceil$, self)$]^{\cdot}$

[metafree ($\lceil\Pi\mathsf{c}\colon\Pi\mathsf{d}\colon\mathsf{a}\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\Pi\mathsf{c}\colon(\mathsf{c}\vdash\mathsf{b})\rceil$, self)$]^{\cdot}$

[metafree ($\lceil\Pi\mathsf{c}\colon\Pi\mathsf{b}\colon\mathsf{a}\rceil$, $\lceil\mathsf{a}\rceil$, $\lceil\Pi\mathsf{c}\colon(\mathsf{c}\vdash\mathsf{b})\rceil$, self)$]^{-}$

$[\mathrm{metafree}\ (\ \lceil \mathsf{a}+\mathsf{b}\rceil\ ,\ \lceil\mathsf{a}\rceil\ ,\ \lceil x+y\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{metafree}\ (\ \lceil \mathsf{a}+\mathsf{b}\rceil\ ,\ \lceil\mathsf{a}\rceil\ ,\ \lceil x\vdash y\rceil\ ,\ \mathrm{self}\ )]^{-}$

$[\mathrm{metafree}\ (\ \lceil \mathsf{c}+\mathsf{b}\rceil\ ,\ \lceil\mathsf{a}\rceil\ ,\ \lceil x\vdash y\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{objectfree}\ (\ \lceil a\rceil\ ,\ \lceil a\rceil\ ,\ \lceil a\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{objectfree}\ (\ \lceil a+b\rceil\ ,\ \lceil a\rceil\ ,\ \lceil \lambda c.c+b\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{objectfree}\ (\ \lceil \lambda a.(a+b)\rceil\ ,\ \lceil a\rceil\ ,\ \lceil \lambda c.c+b\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{objectfree}\ (\ \lceil \lambda b.a\rceil\ ,\ \lceil a\rceil\ ,\ \lceil \lambda c.c+b\rceil\ ,\ \mathrm{self}\ )]^{-}$

$[\mathrm{objectfree}\ (\ \lceil \lambda b.b\rceil\ ,\ \lceil a\rceil\ ,\ \lceil \lambda c.c+b\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{objectfree}\ (\ \lceil \lambda c.a\rceil\ ,\ \lceil a\rceil\ ,\ \lceil \lambda c.c+b\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{objectfree}\ (\ \lceil \lambda c.\lambda d.a\rceil\ ,\ \lceil a\rceil\ ,\ \lceil \lambda c.c+b\rceil\ ,\ \mathrm{self}\ )]^{\cdot}$

$[\mathrm{objectfree}\ (\ \lceil \lambda c.\lambda b.a\rceil\ ,\ \lceil a\rceil\ ,\ \lceil \lambda c.c+b\rceil\ ,\ \mathrm{self}\ )]^{-}$

18.8 Substitution

$[\mathrm{metasubst}\ (\ \lceil\mathsf{a}\rceil\ ,\ \langle\lceil\mathsf{a}\rceil::\lceil\mathsf{b}\rceil\rangle\ ,\ \mathrm{self}\ )\stackrel{\mathrm{t}}{=}\lceil\mathsf{b}\rceil]^{\cdot}$

$[\mathrm{metasubst}\ (\ \lceil\mathsf{a}\rceil\ ,\ \langle\lceil\mathsf{c}\rceil::\lceil\mathsf{b}\rceil\rangle\ ,\ \mathrm{self}\ )\stackrel{\mathrm{t}}{=}\lceil\mathsf{a}\rceil]^{\cdot}$

$[\mathrm{metasubst}\ (\ \lceil\Pi\mathsf{a}\colon\mathsf{a}\rceil\ ,\ \langle\lceil\mathsf{a}\rceil::\lceil\mathsf{b}\rceil\rangle\ ,\ \mathrm{self}\ )\stackrel{\mathrm{t}}{=}\lceil\Pi\mathsf{a}\colon\mathsf{a}\rceil]^{\cdot}$

$[\mathrm{metasubst}\ (\ \lceil\Pi\mathsf{b}\colon\mathsf{a}\rceil\ ,\ \langle\lceil\mathsf{a}\rceil::\lceil\mathsf{b}\rceil\rangle\ ,\ \mathrm{self}\ )\stackrel{\mathrm{t}}{=}\lceil\Pi\mathsf{b}\colon\mathsf{b}\rceil]^{\cdot}$

$[\mathrm{metasubst}\ (\ \lceil\mathsf{a}\vdash\mathsf{c}\rceil\ ,\ \langle\lceil\mathsf{a}\rceil::\lceil\mathsf{b}\rceil\rangle\ ,\ \mathrm{self}\ )\stackrel{\mathrm{t}}{=}\lceil\mathsf{b}\vdash\mathsf{c}\rceil]^{\cdot}$

18.9 Sequent operators

$[\mathrm{eval\text{-}Init}\ (\ \lceil 1\rceil\ ,\ \mathrm{self}\ )\ \mathrm{sequent}{=}\ \langle\langle\lceil 1\rceil\rangle,\langle\,\rangle,\lceil 1\rceil\rangle]^{\cdot}$

$[\mathrm{eval\text{-}Init}\ (\ \lceil 1+\mathsf{a}\rceil\ ,\ \mathrm{self}\ )\ \mathrm{sequent}{=}\ \langle\langle\lceil 1+\mathsf{a}\rceil\rangle,\langle\,\rangle,\lceil 1+\mathsf{a}\rceil\rangle]^{\cdot}$

$[\mathrm{eval\text{-}Init}\ (\ \lceil 1+(\mathsf{a}\vdash\mathsf{a})\rceil\ ,\ \mathrm{self}\ )^{\circ\mathrm{h}}]^{\cdot}$

$[\mathrm{eval\text{-}Ponens}\ (\ \langle\langle\lceil 1\rceil\rangle,\langle\lceil 2\rceil\rangle,\lceil 3\vdash 4\rceil\rangle\ ,\ \lceil\mathsf{T}\rceil\ ,\ \mathrm{self}\ )\ \mathrm{sequent}{=}\ \langle\langle\lceil 3\rceil,\lceil 1\rceil\rangle,\langle\lceil 2\rceil\rangle,\lceil 4\rceil\rangle]^{\cdot}$

$[\mathrm{eval\text{-}Ponens}\ (\ \langle\langle\lceil 1\rceil\rangle,\langle\lceil 2\rceil\rangle,\lceil 3\Vdash 4\rceil\rangle\ ,\ \lceil\mathsf{T}\rceil\ ,\ \mathrm{self}\ )^{\circ\mathrm{h}}]^{\cdot}$

$[\mathrm{eval\text{-}Probans}\ (\ \langle\langle\lceil 1\rceil\rangle,\langle\lceil 2\rceil\rangle,\lceil 3\Vdash 4\rceil\rangle\ ,\ \lceil\mathsf{T}\rceil\ ,\ \mathrm{self}\ )\ \mathrm{sequent}{=}\ \langle\langle\lceil 1\rceil\rangle,\langle\lceil 3\rceil,\lceil 2\rceil\rangle,\lceil 4\rceil\rangle]^{\cdot}$

$[\mathrm{eval\text{-}Probans}\ (\ \langle\langle\lceil 1\rceil\rangle,\langle\lceil 2\rceil\rangle,\lceil 3\vdash 4\rceil\rangle\ ,\ \lceil\mathsf{T}\rceil\ ,\ \mathrm{self}\ )^{\circ\mathrm{h}}]^{\cdot}$

$[\mathrm{eval\text{-}Verify}\ (\ \langle\langle\,\rangle,\langle\,\rangle,\lceil\lambda c.2+2=4\Vdash 5\rceil\rangle\ ,\ \lceil\mathsf{T}\rceil\ ,\ \mathrm{self}\ )\ \mathrm{sequent}{=}\ \langle\langle\,\rangle,\langle\,\rangle,\lceil 5\rceil\rangle]^{\cdot}$

$[\mathrm{eval\text{-}Verify}\ (\ \langle\langle\,\rangle,\langle\,\rangle,\lceil\lambda c.2+2=5\Vdash 5\rceil\rangle\ ,\ \lceil\mathsf{T}\rceil\ ,\ \mathrm{self}\ )^{\circ\mathrm{h}}]^{\cdot}$

[eval-Curry ($\langle\langle\rangle,\langle\rangle,\lceil(1\oplus 2)\vdash 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self) sequent= $\langle\langle\rangle,\langle\rangle,\lceil 1\vdash 2\vdash 3\rceil\rangle]^{\cdot}$

[eval-Curry ($\langle\langle\rangle,\langle\rangle,\lceil(1\vdash 2)\vdash 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-Uncurry ($\langle\langle\rangle,\langle\rangle,\lceil 1\vdash 2\vdash 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self) sequent= $\langle\langle\rangle,\langle\rangle,\lceil(1\oplus 2)\vdash 3\rceil\rangle]^{\cdot}$

[eval-Uncurry ($\langle\langle\rangle,\langle\rangle,\lceil 1\vdash 2\Vdash 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-Deref ($\langle\langle\rangle,\langle\rangle,\lceil\mathrm{A1}\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self) sequent= $\langle\langle\rangle,\langle\rangle,\lceil\Pi\mathsf{x}\colon\Pi\mathsf{y}\colon\mathsf{x}\Rightarrow\mathsf{y}\Rightarrow\mathsf{x}\rceil\rangle]^{\cdot}$

[eval-at ($\lceil(\Pi\mathsf{a}\colon(\mathsf{a}\vdash\mathsf{b}))^{\mathrm{I}}\ @\ \mathsf{a}\rceil$, T , self) sequent= $\langle\langle\lceil\Pi\mathsf{a}\colon(\mathsf{a}\vdash\mathsf{b})\rceil\rangle,\langle\rangle,\lceil\mathsf{a}\vdash\mathsf{b}\rceil\rangle]^{\cdot}$

[eval-at ($\lceil(\Pi\mathsf{a}\colon(\mathsf{a}\vdash\mathsf{b}))^{\mathrm{I}}\ @\ \mathsf{b}\rceil$, T , self) sequent= $\langle\langle\lceil\Pi\mathsf{a}\colon(\mathsf{a}\vdash\mathsf{b})\rceil\rangle,\langle\rangle,\lceil\mathsf{b}\vdash\mathsf{b}\rceil\rangle]^{\cdot}$

[eval-at ($\lceil(\Pi\mathsf{a}\colon(\mathsf{a}\vdash\mathsf{b}))^{\mathrm{I}}\ @\ \mathsf{c}\rceil$, T , self) sequent= $\langle\langle\lceil\Pi\mathsf{a}\colon(\mathsf{a}\vdash\mathsf{b})\rceil\rangle,\langle\rangle,\lceil\mathsf{c}\vdash\mathsf{b}\rceil\rangle]^{\cdot}$

[eval-at ($\lceil(\mathsf{a}\vdash\mathsf{b})^{\mathrm{I}}\ @\ \mathsf{c}\rceil$, T , self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-at ($\lceil(\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon(\mathsf{a}\vdash\mathsf{b}))^{\mathrm{I}}\ @\ \mathsf{a}\rceil$, T , self) sequent= $\langle\langle\lceil\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon(\mathsf{a}\vdash\mathsf{b})\rceil\rangle,\langle\rangle,\lceil\Pi\mathsf{b}\colon\mathsf{a}$
$\mathsf{b}\rceil\rangle]^{\cdot}$

[eval-at ($\lceil(\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon(\mathsf{a}\vdash\mathsf{b}))^{\mathrm{I}}\ @\ \mathsf{b}\rceil$, T , self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-at ($\lceil(\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon(\mathsf{a}\vdash\mathsf{b}))^{\mathrm{I}}\ @\ \mathsf{c}\rceil$, T , self) sequent= $\langle\langle\lceil\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon(\mathsf{a}\vdash\mathsf{b})\rceil\rangle,\langle\rangle,\lceil\Pi\mathsf{b}\colon\mathsf{c}$
$\mathsf{b}\rceil\rangle]^{\cdot}$

[eval-at ($\lceil(\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon(\mathsf{a}\vdash\mathsf{b}))^{\mathrm{I}}\ @\ \mathsf{b}\ @\ \mathsf{a}\rceil$, T , self) sequent= $\langle\langle\lceil\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon(\mathsf{a}\vdash$
$\mathsf{b})\rceil\rangle,\langle\rangle,\lceil\mathsf{b}\vdash\mathsf{a}\rceil\rangle]^{\cdot}$

[eval-Deref ($\langle\langle\rangle,\langle\rangle,\lceil\mathsf{T}\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-infer ($\lceil 2\rceil$, $\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\rangle,\lceil 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self) sequent= $\langle\langle\lceil 1\rceil\rangle,\langle\rangle,\lceil 2\vdash 3\rceil\rangle]^{\cdot}$

[eval-infer ($\lceil 2+\Pi\mathsf{a}\colon\mathsf{a}\rceil$, $\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\rangle,\lceil 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-endorse ($\lceil 2\rceil$, $\langle\langle\rangle,\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\lceil 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self) sequent= $\langle\langle\rangle,\langle\lceil 1\rceil\rangle,\lceil 2\Vdash$
$3\rceil\rangle]^{\cdot}$

[eval-endorse ($\lceil 2+\Pi\mathsf{a}\colon\mathsf{a}\rceil$, $\langle\langle\rangle,\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\lceil 3\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-ie ($\langle\langle\rangle,\langle\rangle,\lceil\Pi\mathsf{x}\colon\Pi\mathsf{y}\colon\mathsf{x}\Rightarrow\mathsf{y}\Rightarrow\mathsf{x}\rceil\rangle$, $\lceil\mathrm{A1}\rceil$, $\lceil\mathsf{T}\rceil$, self) sequent= $\langle\langle\rangle,\langle\rangle,\lceil\mathrm{A1}\rceil\rangle]^{\cdot}$

[eval-ie ($\langle\langle\rangle,\langle\rangle,\lceil\Pi\mathsf{a}\colon\Pi\mathsf{b}\colon\mathsf{a}\Rightarrow\mathsf{b}\Rightarrow\mathsf{a}\rceil\rangle$, $\lceil\mathrm{A2}\rceil$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-all ($\lceil\mathsf{a}\rceil$, $\langle\langle\lceil\mathsf{b}\rceil\rangle,\langle\lceil\mathsf{c}\rceil\rangle,\lceil\mathsf{a}\vdash\mathsf{b}\rceil\rangle$, $\lceil\mathsf{T}\rceil$, self) sequent= $\langle\langle\lceil\mathsf{b}\rceil\rangle,\langle\lceil\mathsf{c}\rceil\rangle,\lceil\Pi\mathsf{a}\colon\mathsf{a}\vdash$
$\mathsf{b}\rceil\rangle]^{\cdot}$

[eval-all ($\langle\langle\lceil\mathsf{a}\rceil\rangle,\langle\rangle,\lceil\mathsf{a}\vdash\mathsf{b}\rceil\rangle$, $\lceil\mathsf{a}\rceil$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-all ($\langle\langle\rangle,\langle\lceil\mathsf{a}\rceil\rangle,\lceil\mathsf{a}\vdash\mathsf{b}\rceil\rangle$, $\lceil\mathsf{a}\rceil$, $\lceil\mathsf{T}\rceil$, self $)^{\circ\mathrm{h}}]^{\cdot}$

[eval-cut ($\langle\langle\lceil 1\rceil,\lceil 2\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil\rangle,\lceil 5\rceil\rangle$, $\langle\langle\lceil 6\rceil,\lceil 5\rceil\rangle,\langle\lceil 7\rceil,\lceil 8\rceil\rangle,\lceil 9\rceil\rangle$, self) sequent=
$\langle\langle\lceil 1\rceil,\lceil 2\rceil,\lceil 6\rceil\rangle,\langle\lceil 3\rceil,\lceil 4\rceil,\lceil 7\rceil,\lceil 8\rceil\rangle,\lceil 9\rceil\rangle]^{\cdot}$

18.10 Verification

$[\text{claiming}\ (\ \lceil 1 \rceil\ ,\ \lceil 1 \wedge_c 2 \rceil\ )]^{\cdot}$

$[\text{claiming}\ (\ \lceil 1 \rceil\ ,\ \lceil 2 \wedge_c 1 \rceil\ )]^{\cdot}$

$[\text{claiming}\ (\ \lceil 1 \rceil\ ,\ \lceil 2 \wedge_c 3 \rceil\ )]^{-}$

18.11 Conversions from values to terms

$[\text{int2string}\ (\ \mathsf{T}\ ,\ 0\ ) = \langle 0 :: 0 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ 1\ ) = \langle 0 :: 1 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ 2\ ) = \langle 0 :: 2 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ 5\ ) = \langle 0 :: 5 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ 10\ ) = \langle 0 :: 10 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ 123\ ) = \langle 0 :: 123 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ {}^{-}1\ ) = \langle 0 :: -1 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ {}^{-}2\ ) = \langle 0 :: -2 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ {}^{-}5\ ) = \langle 0 :: -5 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ {}^{-}10\ ) = \langle 0 :: -10 :: \mathsf{T} \rangle]^{=}$

$[\text{int2string}\ (\ \mathsf{T}\ ,\ {}^{-}123\ ) = \langle 0 :: -123 :: \mathsf{T} \rangle]^{=}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ \mathsf{T}\ ) \stackrel{t}{=} \lceil \mathsf{T} \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ \mathsf{F}\ ) \stackrel{t}{=} \lceil \mathsf{F} \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ \mathsf{T} :: \mathsf{F}\ ) \stackrel{t}{=} \lceil \mathsf{T} :: \mathsf{F} \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ \text{map}\ (\ x\ )\ ) \stackrel{t}{=} \lceil \text{map}\ (\ \cdots\ ) \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ 0\ ) \stackrel{t}{=} \lceil 0 \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ 1\ ) \stackrel{t}{=} \lceil 1 \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ 2\ ) \stackrel{t}{=} \lceil 10 \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ -1\ ) \stackrel{t}{=} \lceil -1 \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ -2\ ) \stackrel{t}{=} \lceil -10 \rceil]^{\cdot}$

$[\text{val2term}\ (\ \mathsf{T}\ ,\ \text{object}\ (\ (1 :: 2) :: 3\ )\ ) \stackrel{t}{=} \lceil \text{object}\ (\ (1 :: 10) :: 11\ ) \rceil]^{\cdot}$

18.12 Unification

$[\text{inst} \ (\ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \mathsf{x} \ \vdash \ \mathsf{y} \ \vdash \ 5 \rceil \ , \ \text{self} \ ) \ , \ \lceil \mathsf{T} \rceil \ , \ \mathsf{T}[1 \to \lceil 3 \rceil][2 \to \lceil 4 \rceil] \ ) \stackrel{\text{t}}{=} \lceil \Pi 3\colon \Pi 4\colon 3 \vdash 4 \vdash 5 \rceil]^{\cdot}$

$[\text{unify} \ (\ \lceil 2 \rceil \ , \ \lceil 2 \rceil \ , \ \mathsf{T} \ )^{\circ \text{h}}]^{-}$

$[\text{unify} \ (\ \lceil 2 \rceil \ , \ \lceil 3 \rceil \ , \ \mathsf{T} \ )^{\circ \text{h}}]^{\cdot}$

$[\text{unify} \ (\ \lceil 2+3 \rceil \ , \ \lceil 2+3 \rceil \ , \ \mathsf{T} \ )^{\circ \text{h}}]^{-}$

$[\text{unify} \ (\ \lceil 2+3 \rceil \ , \ \lceil 2 \cdot 3 \rceil \ , \ \mathsf{T} \ )^{\circ \text{h}}]^{\cdot}$

$[\text{unify} \ (\ \lceil 2+3 \rceil \ , \ \lceil 3+3 \rceil \ , \ \mathsf{T} \ )^{\circ \text{h}}]^{\cdot}$

$[\text{unify} \ (\ \lceil 2+3 \rceil \ , \ \lceil 2+2 \rceil \ , \ \mathsf{T} \ )^{\circ \text{h}}]^{\cdot}$

$[\text{unify} \ (\ \lceil \text{unifresh} \ (\ \lceil v \rceil \ , \ 1 \ ) \rceil \ , \ \lceil 2 \rceil \ , \ \mathsf{T} \ )[1] \stackrel{\text{t}}{=} \lceil 2 \rceil]^{\cdot}$

$[\text{unify} \ (\ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \mathsf{x} \rceil \ , \ \text{self} \ )^{2} \ , \ \lceil 2 \rceil \ , \ \mathsf{T} \ )[1] \stackrel{\text{t}}{=} \lceil 2 \rceil]^{\cdot}$

$[\text{unify} \ (\ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \mathsf{x} :: \mathsf{x} :: \mathsf{y} :: \mathsf{y} \rceil \ , \ \text{self} \ )^{22} \ , \ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \Pi \mathsf{u}\colon \Pi \mathsf{v}\colon 2 :: \mathsf{u} :: \mathsf{u} :$
$, \ \text{self} \ )^{2222} \ , \ \mathsf{T} \ )[1] \stackrel{\text{t}}{=} \lceil 2 \rceil]^{\cdot}$

$[\text{unify} \ (\ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \mathsf{x} :: \mathsf{x} :: \mathsf{y} :: \mathsf{y} \rceil \ , \ \text{self} \ )^{22} \ , \ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \Pi \mathsf{u}\colon \Pi \mathsf{v}\colon 2 :: \mathsf{u} :: \mathsf{u} :$
$, \ \text{self} \ )^{2222} \ , \ \mathsf{T} \ )[2] \stackrel{\text{t}}{=} \lceil \text{unifresh} \ (\ \mathsf{u} \ , \ 3 \ ) \rceil]^{\cdot}$

$[\text{unify} \ (\ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \mathsf{x} :: \mathsf{x} :: \mathsf{y} :: \mathsf{y} \rceil \ , \ \text{self} \ )^{22} \ , \ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \Pi \mathsf{u}\colon \Pi \mathsf{v}\colon 2 :: \mathsf{u} :: \mathsf{u} :$
$, \ \text{self} \ )^{2222} \ , \ \mathsf{T} \ )[3] \stackrel{\text{t}}{=} \lceil 2 \rceil]^{\cdot}$

$[\text{unify} \ (\ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \mathsf{x} :: \mathsf{x} :: \mathsf{y} :: \mathsf{y} \rceil \ , \ \text{self} \ )^{22} \ , \ \text{pterm} \ (\ \lceil \Pi \mathsf{x}\colon \Pi \mathsf{y}\colon \Pi \mathsf{u}\colon \Pi \mathsf{v}\colon 2 :: \mathsf{u} :: \mathsf{u} :$
$, \ \text{self} \ )^{2222} \ , \ \mathsf{T} \ )[4] \stackrel{\text{t}}{=} \lceil 2 \rceil]^{\cdot}$

18.13 Testing of rack functions

$[\text{sl2rack} \ (\ \text{bt2vector}* \ (\ 3 \ ) \ ) = \mathsf{T}]^{=}$

$[\text{sl2rack} \ (\ \text{bt2vector}* \ (\ 1 :: 1 :: 4 \ ) \ ) = \mathsf{T} :: \mathsf{T}]^{=}$

$[\text{sl2rack} \ (\ \text{bt2vector}* \ (\ 0 :: 7 :: 4 \ ) \ ) = 7]^{=}$

$[\text{sl2rack} \ (\ \text{bt2vector}* \ (\ 2 :: 3 :: 7 :: 8 :: 9 :: 4 \ ) \ ) = \text{bt2vector} \ (\ 7 :: 8 :: 9 \ )]^{=}$

$[\text{sl2rack} \ (\ \text{bt2vector}* \ (\ 0 :: 6 :: 2 :: 3 :: 7 :: 8 :: 9 :: 3 :: 4 :: 6 \ ) \ ) = 6 :: \text{bt2vector} \ (\ 7 :: 8 :: 9 \ )]^{=}$

$[\text{sl2rack} \ (\ \text{bt2vector}* \ (\ 0 :: 6 :: 2 :: 3 :: 7 :: 8 :: 9 :: 3 :: 4 :: 6 :: 0 \ ) \ ) = 6 :: \text{bt2vector} \ (\ 7 :: 8 :: 9 \ )]^{=}$

$[\text{sl2rack} \ (\ \text{append} \ (\ \text{bt2vector}* \ (\ 0 :: 6 :: 2 :: 3 :: 7 :: 8 :: 9 :: 3 :: 4 :: 6 \ ) \ , \ \langle \mathsf{F} \rangle \ ) \ ) = 6 :: \text{bt2vector} \ (\ 7 :: 8 :: 9 \ )]^{=}$

[sl2rack (append (bt2vector∗ ($0::6::2::3::7::8::9::3::4$) , ⟨F⟩)) = •]$^=$

[sl2rack (bt2vector∗ ($0::6::2::3::7::8::9::3::4$)) = •]$^=$

[bt2vector∗ (⟨3⟩) = rack2sl (T)]$^=$

[bt2vector∗ (⟨1, 1, 4⟩) = rack2sl (T : : T)]$^=$

[bt2vector∗ (⟨0, 7, 4⟩) = rack2sl (7)]$^=$

[bt2vector∗ (⟨2, 3, 7, 8, 9, 4⟩) = rack2sl (bt2vector (7 : : 8 : : 9))]$^=$

[bt2vector∗ (⟨0, 6, 2, 3, 7, 8, 9, 3, 4, 6⟩) = rack2sl (6 : : bt2vector (7 : : 8 : : 9))]$^=$

[bt2vector∗ (⟨0, 7, 3, 3, 5⟩) = rack2sl (7 : : 7)]$^=$

[bt2vector∗ (⟨1, 1, 3, 3, 5⟩) = rack2sl ((T : : T) : : (T : : T))]$^=$

[• = rack2sl ((T : : T) : : (T : : −1))]$^=$

[• = rack2sl ((T : : T) : : (T : : F))]$^=$

18.14 Test of Ripemd

[‘9C1185A5C5E9FC54612808977EE8F548B2258D31’ = ripemds (‘’)]$^=$

[‘0BDC9D2D256B3EE9DAAE347BE6F4DC835A467FFE’ = ripemds (‘a’)]$^=$

[‘8EB208F7E05D987A9B044A8E98C6B087F15A0BFC’ = ripemds (‘abc’)]$^=$

[‘8EB208F7E05D987A9B044A8E98C6B087F15A0BFC’ = ripemds (⟨‘a’, ‘bc’⟩)]$^=$

[‘5D0689EF49D2FAE572B881B123A85FFA21595F36’ = ripemds (‘message digest’)]$^=$

[‘F71C27109C692C1B56BBDCEB5B9D2865B3708DBC’ = ripemds (‘abcdefghijkl)]$^=$

[‘12A053384A9C0C88E405A06C27DCF49ADA62EB2B’ = ripemds (‘abcdbcdecde)]$^=$

[‘B0E20B6E3116640286ED3A87A5713079B21F5189’ = ripemds (‘ABCDEFGHIJI)]$^=$

[‘9B752E45573D4B39F4DBD3323CAB82BF63326BFB’ = ripemds (repeat (8 , ‘1234567890’))]$^=$

18.15 Compilation

$[\text{compiled-base} \stackrel{\text{val}}{\to} \text{norm compile (base)}]$

$[\text{compiled-test} \stackrel{\text{val}}{\to} \text{norm compile (test)}]$

$[\text{get-base0 (} x \text{)} \stackrel{\text{val}}{\to} \text{base}[x^{\text{r}}][\text{code}][x^{\text{i}}]]$

$[\text{get-base1 (} x \text{)} \stackrel{\text{val}}{\to} \text{compiled-base}[x^{\text{r}}][\text{code}][x^{\text{i}}]]$

$[\text{get-test0 (} x \text{)} \stackrel{\text{val}}{\to} \text{test}[x^{\text{r}}][\text{code}][x^{\text{i}}]]$

$[\text{get-test1 (} x \text{)} \stackrel{\text{val}}{\to} \text{compiled-test}[x^{\text{r}}][\text{code}][x^{\text{i}}]]$

$[\text{notnot test[test[0]][expansion]}]^{-}$

$[\text{notnot compile (test)[test[0]][expansion]}]^{-}$

$[\text{notnot (norm compile (test))[test[0]][expansion]}]^{-}$

$[\text{notnot compiled-test[test[0]][expansion]}]^{-}$

$[\text{get-base0 (} \lceil \lambda x.y \rceil \text{)} = 0]^{=}$

$[\text{get-base1 (} \lceil \lambda x.y \rceil \text{)} = 0]^{=}$

$[\text{get-base0 (} \lceil \lceil x \rceil \rceil \text{)} = 1]^{=}$

$[\text{get-base1 (} \lceil \lceil x \rceil \rceil \text{)} = 1]^{=}$

$[\text{get-base0 (} \lceil \mathsf{T} \rceil \text{)} = \text{map (} \mathsf{T} \text{)}]^{=}$

$[\text{get-base1 (} \lceil \mathsf{T} \rceil \text{)} = \text{map (} \mathsf{T} \text{)}]^{=}$

$[\text{get-base0 (} \lceil x \text{ ' } y \rceil \text{)} = \text{map (} \mathsf{T} \text{)}]^{=}$

$[\text{get-base1 (} \lceil x \text{ ' } y \rceil \text{)} = \text{map (} \mathsf{T} \text{)}]^{=}$

$[\text{get-base0 (} \lceil \textbf{If } x \textbf{ then } y \textbf{ else } z \rceil \text{)} = \text{map (} \mathsf{T} \text{)}]^{=}$

$[\text{get-base1 (} \lceil \textbf{If } x \textbf{ then } y \textbf{ else } z \rceil \text{)} = \text{map (} \mathsf{T} \text{)}]^{=}$

$[\text{get-base0 (} \lceil \mathsf{T} \rceil \text{) Tail} = \mathsf{T}]^{=}$

$[\text{get-base1 (} \lceil \mathsf{T} \rceil \text{) Tail} = \mathsf{T}]^{=}$

$[\text{get-base0 (} \lceil x \text{ ' } y \rceil \text{) Tail ' } (\lambda x.x) \text{ ' } 2 = 2]^{=}$

$[\text{get-base1 (} \lceil x \text{ ' } y \rceil \text{) Tail ' } (\lambda x.x) \text{ ' } 2 = 2]^{=}$

$[\text{get-base0 (} \lceil x \text{ ' } y \rceil \text{) Tail ' } (\lambda x.3) \text{ ' } 2 = 3]^{=}$

$[\text{get-base1 (} \lceil x \text{ ' } y \rceil \text{) Tail ' } (\lambda x.3) \text{ ' } 2 = 3]^{=}$

$[\text{get-base0 (}\ulcorner\textbf{If } x \textbf{ then } y \textbf{ else } z\urcorner\text{) Tail ' }\mathsf{T}\text{ ' 1 ' 2} = 1]^{=}$

$[\text{get-base1 (}\ulcorner\textbf{If } x \textbf{ then } y \textbf{ else } z\urcorner\text{) Tail ' }\mathsf{T}\text{ ' 1 ' 2} = 1]^{=}$

$[\text{get-base0 (}\ulcorner\textbf{If } x \textbf{ then } y \textbf{ else } z\urcorner\text{) Tail ' }\mathsf{F}\text{ ' 1 ' 2} = 2]^{=}$

$[\text{get-base1 (}\ulcorner\textbf{If } x \textbf{ then } y \textbf{ else } z\urcorner\text{) Tail ' }\mathsf{F}\text{ ' 1 ' 2} = 2]^{=}$

$[\text{make-constant (}\ulcorner 123\urcorner\text{) Tail ' (2 LazyPair 3) } \stackrel{\text{t}}{=} \ulcorner 123\urcorner]^{\cdot}$

$[\text{make-lambda (make-constant (}\ulcorner 123\urcorner\text{)) Tail ' }\mathsf{T}\text{ ' 4 } \stackrel{\text{t}}{=} \ulcorner 123\urcorner]^{\cdot}$

$[\text{compile-test00} \stackrel{\text{val}}{\to} \ulcorner 123\urcorner]$

$[\text{get-test0 (}\ulcorner\text{compile-test00}\urcorner\text{) Tail } \stackrel{\text{t}}{=} \ulcorner 123\urcorner]^{\cdot}$

$[\text{get-test1 (}\ulcorner\text{compile-test00}\urcorner\text{) Tail } \stackrel{\text{t}}{=} \ulcorner 123\urcorner]^{\cdot}$

$[\text{compile-test01 (} x \text{)} \stackrel{\text{val}}{\to} \ulcorner 123\urcorner]$

$[\text{get-test0 (}\ulcorner\text{compile-test01 (} x \text{)}\urcorner\text{) Tail ' 9 } \stackrel{\text{t}}{=} \ulcorner 123\urcorner]^{\cdot}$

$[\text{get-test1 (}\ulcorner\text{compile-test01 (} x \text{)}\urcorner\text{) Tail ' 9 } \stackrel{\text{t}}{=} \ulcorner 123\urcorner]^{\cdot}$

$[\text{compile-test02 (} x \text{)} \stackrel{\text{val}}{\to} x]$

$[\text{get-test0 (}\ulcorner\text{compile-test02 (} x \text{)}\urcorner\text{) Tail ' 9} = 9]^{=}$

$[\text{get-test1 (}\ulcorner\text{compile-test02 (} x \text{)}\urcorner\text{) Tail ' 9} = 9]^{=}$

$[\text{compile-test03 (} x \text{ , } y \text{)} \stackrel{\text{val}}{\to} x]$

$[\text{get-test0 (}\ulcorner\text{compile-test03 (} x \text{ , } y \text{)}\urcorner\text{) Tail ' 8 ' 9} = 8]^{=}$

$[\text{get-test1 (}\ulcorner\text{compile-test03 (} x \text{ , } y \text{)}\urcorner\text{) Tail ' 8 ' 9} = 8]^{=}$

$[\text{compile-test04 (} x \text{ , } y \text{)} \stackrel{\text{val}}{\to} y]$

$[\text{get-test0 (}\ulcorner\text{compile-test04 (} x \text{ , } y \text{)}\urcorner\text{) Tail ' 8 ' 9} = 9]^{=}$

$[\text{get-test1 (}\ulcorner\text{compile-test04 (} x \text{ , } y \text{)}\urcorner\text{) Tail ' 8 ' 9} = 9]^{=}$

$[\text{compile-test05 (} x \text{ , } y \text{)} \stackrel{\text{val}}{\to} z]$

$[\text{get-test0 (}\ulcorner\text{compile-test05 (} x \text{ , } y \text{)}\urcorner\text{) Tail ' 8 ' 9} = \mathsf{T}]^{=}$

$[\text{get-test1 (}\ulcorner\text{compile-test05 (} x \text{ , } y \text{)}\urcorner\text{) Tail ' 8 ' 9} = \mathsf{T}]^{=}$

$[\text{compile-test06 (} x \text{ , } x \text{)} \stackrel{\text{val}}{\to} x]$

$[\text{get-test0 (}\ulcorner\text{compile-test06 (} x \text{ , } y \text{)}\urcorner\text{) Tail ' 8 ' 9} = 8]^{=}$

$[\text{get-test1} (\lceil \text{compile-test06} (x , y) \rceil) \text{ Tail} ' 8 ' 9 = 8]^{=}$

$[\text{compile-test07} \stackrel{\text{val}}{\to} \text{abc}]$

$[\text{get-test0} (\lceil \text{compile-test07} \rceil) \text{ Tail} = \text{abc}]^{=}$

$[\text{get-test1} (\lceil \text{compile-test07} \rceil) \text{ Tail} = \text{abc}]^{=}$

$[\text{compile-test08} \stackrel{\text{val}}{\to} \text{test}]$

$[\text{get-test0} (\lceil \text{compile-test08} \rceil) \text{ Tail}[0] = \lceil \text{test} \rceil^{\text{r}}]^{=}$

$[\text{get-test1} (\lceil \text{compile-test08} \rceil) \text{ Tail}[0] = \lceil \text{test} \rceil^{\text{r}}]^{=}$

$[\text{compile-test09} \stackrel{\text{val}}{\to} \text{base}]$

$[\text{get-test0} (\lceil \text{compile-test09} \rceil) \text{ Tail}[0] = \lceil \text{base} \rceil^{\text{r}}]^{=}$

$[\text{get-test1} (\lceil \text{compile-test09} \rceil) \text{ Tail}[0] = \lceil \text{base} \rceil^{\text{r}}]^{=}$

$[\text{compile-test10} (u , v) \stackrel{\text{val}}{\to} \lambda x. \lambda y. u]$

$[\text{get-test0} (\lceil \text{compile-test10} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 1]^{=}$

$[\text{get-test1} (\lceil \text{compile-test10} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 1]^{=}$

$[\text{compile-test11} (u , v) \stackrel{\text{val}}{\to} \lambda x. \lambda y. v]$

$[\text{get-test0} (\lceil \text{compile-test11} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 2]^{=}$

$[\text{get-test1} (\lceil \text{compile-test11} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 2]^{=}$

$[\text{compile-test12} (u , v) \stackrel{\text{val}}{\to} \lambda x. \lambda y. x]$

$[\text{get-test0} (\lceil \text{compile-test12} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 3]^{=}$

$[\text{get-test1} (\lceil \text{compile-test12} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 3]^{=}$

$[\text{compile-test13} (u , v) \stackrel{\text{val}}{\to} \lambda x. \lambda y. y]$

$[\text{get-test0} (\lceil \text{compile-test13} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 4]^{=}$

$[\text{get-test1} (\lceil \text{compile-test13} (x , y) \rceil) \text{ Tail} ' 1 ' 2 ' 3 ' 4 = 4]^{=}$

$[\text{compile-test14} \stackrel{\text{val}}{\to} \text{Base}]$

$[\text{get-test0} (\lceil \text{compile-test14} \rceil) \text{ Tail} = 10]^{=}$

$[\text{get-test1} (\lceil \text{compile-test14} \rceil) \text{ Tail} = 10]^{=}$

$[\text{compile-test15} (\text{Base} , x) \stackrel{\text{val}}{\to} \text{Base}]$

$[\text{get-test0} (\lceil \text{compile-test15} (x , y) \rceil) \text{ Tail} ' 1 ' 2 = 10]^{=}$

$[\text{get-test1 (}\lceil\text{compile-test15 (} x \text{ , } y \text{)}\rceil\text{) Tail ' 1 ' 2} = 10]^{=}$

$[\text{compile-test16 (Base , } x \text{)} \overset{\text{val}}{\to} x]$

$[\text{get-test0 (}\lceil\text{compile-test16 (} x \text{ , } y \text{)}\rceil\text{) Tail ' 1 ' 2} = 2]^{=}$

$[\text{get-test1 (}\lceil\text{compile-test16 (} x \text{ , } y \text{)}\rceil\text{) Tail ' 1 ' 2} = 2]^{=}$

$[\text{compile-test17} \overset{\text{val}}{\to} 123]$

$[\text{get-test0 (}\lceil\text{compile-test17}\rceil\text{) Tail} = 123]^{=}$

$[\text{get-test1 (}\lceil\text{compile-test17}\rceil\text{) Tail} = 123]^{=}$

$[\text{compile-test18} \overset{\text{val}}{\to} 123 - 23]$

$[\text{get-test0 (}\lceil\text{compile-test18}\rceil\text{) Tail} = 100]^{=}$

$[\text{get-test1 (}\lceil\text{compile-test18}\rceil\text{) Tail} = 100]^{=}$

$[\text{compile-test19 (} x \text{ , } y \text{)} \overset{\text{val}}{\to} \textbf{if } x \textbf{ then } y \textbf{ else } \text{compile-test19 (} x^{\text{t}} \text{ , } x^{\text{h}} :: y \text{)}]$

$[\langle 3,4\rangle = (\text{get-test0 (}\lceil\text{compile-test19 (} x \text{ , } y \text{)}\rceil\text{) '' } \mathsf{T}^{\text{M}} \text{ '' } \langle 3,4\rangle^{\text{M}})^{\text{U}}]^{=}$

$[\langle 3,4\rangle = (\text{get-test1 (}\lceil\text{compile-test19 (} x \text{ , } y \text{)}\rceil\text{) '' } \mathsf{T}^{\text{M}} \text{ '' } \langle 3,4\rangle^{\text{M}})^{\text{U}}]^{=}$

$[\langle 1,2,3,4\rangle = (\text{get-test0 (}\lceil\text{compile-test19 (} x \text{ , } y \text{)}\rceil\text{) '' } \langle 2,1\rangle^{\text{M}} \text{ '' } \langle 3,4\rangle^{\text{M}})^{\text{U}}]^{=}$

$[\langle 1,2,3,4\rangle = (\text{get-test1 (}\lceil\text{compile-test19 (} x \text{ , } y \text{)}\rceil\text{) '' } \langle 2,1\rangle^{\text{M}} \text{ '' } \langle 3,4\rangle^{\text{M}})^{\text{U}}]^{=}$

$[\mathsf{T} = (\text{get-base0 (}\lceil\mathsf{T}\rceil\text{)) Tail}]^{=}$

$[\mathsf{T} = (\text{get-base1 (}\lceil\mathsf{T}\rceil\text{)) Tail}]^{=}$

$[\mathsf{F} = (\text{get-base0 (}\lceil\mathsf{F}\rceil\text{)) Tail}]^{=}$

$[\mathsf{F} = (\text{get-base1 (}\lceil\mathsf{F}\rceil\text{)) Tail}]^{=}$

$[\text{2 LazyPair 3 Equal get-base0 (}\lceil x \text{ LazyPair } y\rceil\text{) Tail ' 2 ' 3}]^{\cdot}$

$[\text{2 LazyPair 3 Equal get-base1 (}\lceil x \text{ LazyPair } y\rceil\text{) Tail ' 2 ' 3}]^{\cdot}$

$[\text{3 LazyPair 2 Equal get-base0 (}\lceil x \text{ LazyPair } y\rceil\text{) Tail ' 2 ' 3 Tail}]^{-}$

$[\text{3 LazyPair 2 Equal get-base1 (}\lceil x \text{ LazyPair } y\rceil\text{) Tail ' 2 ' 3 Tail}]^{-}$

$[\text{2 LazyPair 3 Equal get-base0 (}\lceil x \text{ Pair } y\rceil\text{) Tail ' 2 ' 3}]^{\cdot}$

$[\text{2 LazyPair 3 Equal get-base1 (}\lceil x \text{ Pair } y\rceil\text{) Tail ' 2 ' 3}]^{\cdot}$

$[\text{get-base0 (}\lceil x \text{ Equal } y\rceil\text{) Tail ' 2 ' 2}]^{\cdot}$

$[\text{get-base1 (}\lceil x \text{ Equal } y\rceil\text{) Tail ' 2 ' 2}]^{\cdot}$

[get-base0 (⌈x Equal y⌉) Tail ’ 2 ’ 3^{M}]$^{-}$

[get-base1 (⌈x Equal y⌉) Tail ’ 2 ’ 3^{M}]$^{-}$

[10 LazyPair T Equal get-base0 (⌈x Pair y⌉) Tail ’ 10 ’ T]$^{\cdot}$

[10 LazyPair T Equal get-base1 (⌈x Pair y⌉) Tail ’ 10 ’ T]$^{\cdot}$

[10 LazyPair F Equal get-base0 (⌈x Pair y⌉) Tail ’ 10 ’ F]$^{\cdot}$

[10 LazyPair F Equal get-base1 (⌈x Pair y⌉) Tail ’ 10 ’ F]$^{\cdot}$

[T LazyPair T Equal get-base0 (⌈x Pair y⌉) Tail ’ T ’ T]$^{\cdot}$

[T LazyPair T Equal get-base1 (⌈x Pair y⌉) Tail ’ T ’ T]$^{\cdot}$

[T LazyPair 10 Equal get-base0 (⌈x Pair y⌉) Tail ’ T ’ 10]$^{\cdot}$

[T LazyPair 10 Equal get-base1 (⌈x Pair y⌉) Tail ’ T ’ 10]$^{\cdot}$

[F LazyPair 10 Equal get-base0 (⌈x Pair y⌉) Tail ’ F ’ 10]$^{\cdot}$

[F LazyPair 10 Equal get-base1 (⌈x Pair y⌉) Tail ’ F ’ 10]$^{\cdot}$

[T LazyPair F Equal get-base0 (⌈x Pair y⌉) Tail ’ T ’ F]$^{\cdot}$

[T LazyPair F Equal get-base1 (⌈x Pair y⌉) Tail ’ T ’ F]$^{\cdot}$

[F LazyPair T Equal get-base0 (⌈x Pair y⌉) Tail ’ F ’ T]$^{\cdot}$

[F LazyPair T Equal get-base1 (⌈x Pair y⌉) Tail ’ F ’ T]$^{\cdot}$

[Zero Equal (get-base0 (⌈Zero⌉)) Tail]$^{\cdot}$

[Zero Equal (get-base1 (⌈Zero⌉)) Tail]$^{\cdot}$

[One Equal (get-base0 (⌈x Pair y⌉) ” map (F) ” map (Zero)) Tail]$^{\cdot}$

[One Equal (get-base1 (⌈x Pair y⌉) ” map (F) ” map (Zero)) Tail]$^{\cdot}$

[One Equal (get-base0 (⌈One⌉)) Tail]$^{\cdot}$

[One Equal (get-base1 (⌈One⌉)) Tail]$^{\cdot}$

[Two Equal (get-base0 (⌈Two⌉)) Tail]$^{\cdot}$

[Two Equal (get-base1 (⌈Two⌉)) Tail]$^{\cdot}$

[Five Equal (get-base0 (⌈x Plus y⌉) ” map (Three) ” map (Two)) Tail]$^{\cdot}$

[Five Equal (get-base1 (⌈x Plus y⌉) ” map (Three) ” map (Two)) Tail]$^{\cdot}$

[10 = (get-base0 (⌈Base⌉))$^{\mathrm{U}}$]$^{=}$

[10 = (get-base1 (⌈Base⌉))$^{\mathrm{U}}$]$^{=}$

$[\langle 1,2,3\rangle = (\text{get-base0}\ (\ \ulcorner\text{revappend}\ (\ x\ ,\ y\ )\urcorner\ )\ \text{''}\ \langle 1\rangle^{\text{M}}\ \text{''}\ \langle 2,3\rangle^{\text{M}})^{\text{U}}]^{=}$

$[\langle 1,2,3\rangle = (\text{get-base1}\ (\ \ulcorner\text{revappend}\ (\ x\ ,\ y\ )\urcorner\ )\ \text{''}\ \langle 1\rangle^{\text{M}}\ \text{''}\ \langle 2,3\rangle^{\text{M}})^{\text{U}}]^{=}$

$[\text{prune}\ (\ \ulcorner 1+2\urcorner\ ,\ \text{test}\ ) \stackrel{\text{t}}{=} \ulcorner 1+2\urcorner]^{\cdot}$

$[\text{prune}\ (\ \ulcorner 1+2\urcorner\ ,\ \text{base}\ ) \stackrel{\text{t}}{=} \ulcorner 1+2\urcorner]^{\cdot}$

$[\text{prune}\ (\ \ulcorner 1+\text{compile-test00}\urcorner\ ,\ \text{test}\ ) \stackrel{\text{t}}{=} \ulcorner 1+\text{compile-test00}\urcorner]^{\cdot}$

$[\text{prune}\ (\ \ulcorner 1+\text{compile-test00}\urcorner\ ,\ \text{base}\ ) \stackrel{\text{t}}{=} \ulcorner 1+\text{base}\urcorner]^{\cdot}$

$[\text{prune}\ (\ \ulcorner\text{compile-test03}\ (\ 1\ ,\ 2\ )\urcorner\ ,\ \text{test}\ ) \stackrel{\text{t}}{=} \ulcorner\text{compile-test03}\ (\ 1\ ,\ 2\ )\urcorner]^{\cdot}$

$[\text{prune}\ (\ \ulcorner\text{compile-test03}\ (\ 1\ ,\ 2\ )\urcorner\ ,\ \text{base}\ ) \stackrel{\text{t}}{=} \ulcorner\text{base}\urcorner]^{\cdot}$

$[\text{compiled-base}[\text{base}[0]][\text{diagnose}] = \mathsf{T}^{\text{M}}]^{=}$

$[\text{compile}\ (\ \text{test}[\langle\text{test}[0],\text{expansion}\rangle\Rightarrow\ulcorner\text{test}\urcorner]\ )[\text{test}[0]][\text{diagnose}] = \mathsf{T}^{\text{M}}]^{\cdot}$

$[\text{compile}\ (\ \text{test}[\langle\text{test}[0],\text{expansion}\rangle\Rightarrow\ulcorner\text{test}\urcorner]\ )[\text{test}[0]][\text{diagnose}]^{\text{U}} = \mathsf{T}]^{\cdot}$

$[\text{compile}\ (\ \text{test}[\langle\text{test}[0],\text{expansion}\rangle\Rightarrow\ulcorner[2+3=5]^{\cdot}\ \text{.then.}\ [2+4=6]^{\cdot}\urcorner]\ )[\text{test}[0]][\text{diagnose}]^{\text{U}} = \mathsf{T}]^{\cdot}$

$[\text{compile}\ (\ \text{test}[\langle\text{test}[0],\text{expansion}\rangle\Rightarrow\ulcorner[2+3=0]^{\cdot}\ \text{.then.}\ [2+4=6]^{\cdot}\urcorner]\ )[\text{test}[0]][\text{diagnose}]^{\text{U11}} \stackrel{\text{t}}{=} \ulcorner[2+3=0]^{\cdot}\urcorner]^{\cdot}$

$[\text{compile}\ (\ \text{test}[\langle\text{test}[0],\text{expansion}\rangle\Rightarrow\ulcorner[2+3=5]^{\cdot}\ \text{.then.}\ [2+4=0]^{\cdot}\urcorner]\ )[\text{test}[0]][\text{diagnose}]^{\text{U11}} \stackrel{\text{t}}{=} \ulcorner[2+4=0]^{\cdot}\urcorner]^{\cdot}$

$[\text{compile}\ (\ \text{test}[\langle\text{test}[0],\text{expansion}\rangle\Rightarrow\ulcorner[2+3=0]^{\cdot}\ \text{.then.}\ [2+4=0]^{\cdot}\urcorner]\ )[\text{test}[0]][\text{diagnose}]^{\text{U11}} \stackrel{\text{t}}{=} \ulcorner[2+3=0]^{\cdot}\urcorner]^{\cdot}$

$[\text{test} \stackrel{\text{val}}{\to} 2+3]$

$[\text{test}[\text{test}[0]][\text{`codex'}][\ulcorner\text{test}\urcorner^{\text{r}}][\ulcorner\text{test}\urcorner^{\text{i}}][0][\text{`value'}] \stackrel{\text{t}}{=} \ulcorner[\text{test} \stackrel{\text{val}}{\to} 2+3]\urcorner]^{\cdot}$

$[\text{get-test0}\ (\ \ulcorner\text{test}\urcorner\ )\ \text{Tail} = 5]^{=}$

$[\text{get-test1}\ (\ \ulcorner\text{test}\urcorner\ )\ \text{Tail} = 5]^{=}$